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GUSSET PLATE EVALUATION GUIDE

Refined Analysis Methods

Developed for Use with the 2014 Interim Revisions to the Manual for Bridge Evaluation, Second Edition, 2010

Illinois Department of Transportation

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Prepared For:

Illinois Department of Transportation
Bureau of Bridge and Structures

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Bureau of Bridges and Structures
Structural Services Section
2300 South Dirksen Parkway
Springfield, Illinois 62764

Prepared by:

WJE

Wiss, Janney, Elstner Associates, Inc.
330 Pfingsten Road
Northbrook, Illinois 60062
847.272.7400 tel | 847.291.4813 fax

Authors:

Howard J. Hill, P.E., S.E., PhD	hhill@wje.com
Steven L. Lauer	slauer@wje.com
Jonathan C. McGormley, P.E., S.E.	jmcgormley@wje.com

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INTRODUCTION

Purpose

The *2014 AASHTO Design Specifications* (Specs) contain formulas for determining the capacities of truss gusset plates. The formulas for determining the ability of a gusset plate to sustain the demands applied by compression web elements were based largely on a study of gusset plate characteristics that was conducted by the FHWA (NCHRP Study). The AASHTO formulas are straightforward, easy to apply, provide conservative estimates of plate capacities, and are consistent with contemporary AASHTO objectives concerning structural reliability. As such, they can be used to efficiently provide reasonable designs for new gusset plates, in most instances.

The AASHTO compression capacity formulas can also be used to get quick estimates of the capacities of existing gusset plates. For this reason, they are included in the *2014 Interim Revisions to the Manual of Bridge Evaluation, Second Edition* (MBE). However, it is very important to note that, when evaluating an existing structure, the cost of being conservative is much greater than when designing a new structure. In the latter case, underestimating the capacity of a gusset plate by 20 percent results in just the added costs of unneeded plate thickness, which is typically a small fraction of the overall cost of designing and building the connection. In the case of an existing bridge, however, the same degree of conservatism can result in the expenditure of tens of thousands of dollars to retrofit a single plate that actually needs no modification. On an entire bridge, construction costs due solely to using very conservative plate capacity estimates can reach millions, to say nothing of the “costs” associated with unneeded disruptions, detours and posted load limits. Therefore, when the standard formulas for plate compression strength indicate a deficiency, it is often worthwhile to perform a more rigorous analysis to eliminate some of the conservatism inherent in the standard approach.

Recognizing the fact that the cost of conservatism can be quite high when dealing with existing structures, the authors of the MBE included text that allows the use of alternative methods for determining gusset plate capacities. The only alternate method discussed explicitly in the MBE involves the development of robust finite element models. While finite element (FE) methods can be used to provide reasonable estimates of gusset plate strengths, the cost of useful FE analyses can quickly approach the cost of retrofitting a gusset plate, and obtaining reliable results requires substantial modeling and analytical expertise.

Fortunately, much of the conservatism inherent in the MBE gusset plate compression checks can be eliminated simply by using enhanced “hand” calculations. Therefore, when the MBE compression checks indicate a deficiency, or if they control a load rating, they should be supplemented by a more rigorous evaluation.

Compared to the basic MBE formulas, the methods outlined in this Guide for calculating plate compression capacities provide much better estimates of actual strengths and, when used in conjunction with the MBE load and strength reduction factors, they result in levels of reliability comparable to that which is targeted by the methods specified in the MBE. The methods outlined in this Guide are meant to replace the MBE formulas related to Whitmore Buckling and Partial Shear. A modified approach for checking Horizontal Shear is also provided.

Use of the Guide methods, or similarly effective approaches, will greatly reduce the amount of resources spent on unnecessary modifications of existing gusset plates, while maintaining a degree of reliability consistent with applicable design provisions.

Outline

This Guide includes four sections. The first section describes Refined Analysis Methods that can be used to evaluate gusset plates. It includes relevant background information on the subjects of reliability, ductility, and buckling. Following the background information are detailed descriptions of the three strength determination checks that are intended to replace some of the basic checks in the MBE when evaluating gusset plates that are 0.375 inches thick or thicker; the Horizontal Shear check (replaces the MBE Horizontal Shear check), the Basic Corner Check (meant to replace the MBE Whitmore L_{mid} and Partial Shear checks), and the Refined Corner Check (meant to replace the MBE Whitmore L_{mid} and Partial Shear checks in instances where the Basic Corner Check does not provide sufficient capacity). The last item in the Refined Analysis Section is a discussion concerning the evaluation of deteriorated gusset plates.

The second section is a Glossary of Terms used in the calculations. The third section provides six gusset plate load rating examples. The examples cover a variety of realistic circumstances and the application of the Guide's Refined Analysis Methods. The fourth section comprises various appendices that include relevant reference materials.

REFINED ANALYSIS METHODS

Background

The AASHTO method for calculating gusset plate compression capacity involves the application of two "checks"; one based on the Whitmore section and a particular equivalent column length (Whitmore L_{mid}), and one based on a check of the shear stress on a specific portion of the plate in the vicinity of the compression member (Partial Shear). These checks were developed to provide quick, efficient estimates of actual strengths. As noted in the NCHRP Study responsible for developing these checks, they tend to underestimate actual plate strengths, at times by a considerable amount.

Recognizing the value of increased analytical rigor and the challenges of creating robust FE models of most truss connections, methods were developed that can provide more accurate capacity estimates for most compression situations in a few hours, using "hand" calculations. While this is much more time than what is required to perform the basic MBE Whitmore L_{mid} and Partial Shear checks, the benefits often far outweigh the costs. Eliminating a single unnecessary connection modification or reducing the extent of modifications that are needed will usually save much more than the added engineering costs. Reducing the severity or even the need for traffic disruption and load postings are added benefits.

The Guide methods are based on fundamental engineering principles, and came from the hand-based evaluation of the U10 gusset plates that initiated the collapse of the I-35W Bridge. Subsequent FE analyses by multiple parties proved the hand calculations to be very accurate. The several joint tests and the many FE simulations performed as part of the NCHRP Study provided other examples of limit state connection performance that were used to improve and verify the hand-based approach that was used in the I-35W Bridge case.

In the context of the Guide and the NCHRP Study, the "Professional Factor" (PF) is the ratio of an element's actual strength (i.e., the ultimate capacity as determined by testing or the ultimate capacity established by FE analysis) divided by the strength predicted by a simplified analytical method. A PF of one indicates a perfect match, while a PF greater than one indicates the simplified method underestimates actual strength. Figure 1 shows the distribution of PFs for 95 gusset plates included in the NCHRP Study, which are 0.375 inches thick or thicker. One curve shows the PFs obtained using the MBE methods, the other shows the PFs obtained using the Guide methods. The overall average PFs for the two methods are 1.33 (MBE) and 1.10 (Guide), and the standard deviations are 0.14 (MBE) and 0.10 (Guide). In all cases where actual tests were performed, both approaches provided conservative results. On a case-by-case basis, the Guide prediction typically varies from being just as good as, to much better than, the MBE prediction. In many cases, the Guide prediction is so much better that any associated load rating would be significantly higher.

Compared to an evaluation based on the MBE formulas, use of the Guide methods would greatly reduce the amount of false deficiency findings and unnecessary gusset plate modifications.

Figure 1 also shows that there remains a great deal of room for improvement in the Guide methods. One of the more significant reasons for the larger PFs in the Guide methods is the fact that they do not account for strain hardening. Study of the NCHRP Study FE simulations shows that the more compact connections realize considerable strain hardening prior to failure. In fact, this phenomenon caused many NCHRP Study FE connection failures to be characterized as "buckling" failures, while the associated Guide failure mode was horizontal shear. Both characterizations are technically correct. The Guide method correctly notes that a plate consisting of material that does not strain harden would fail via horizontal shear, while the FE model shows that a comparable plate that can strain harden as assumed, would buckle before becoming unstable in shear. As work on the Guide methods continues, a method for appropriately accounting for strain hardening may be developed.

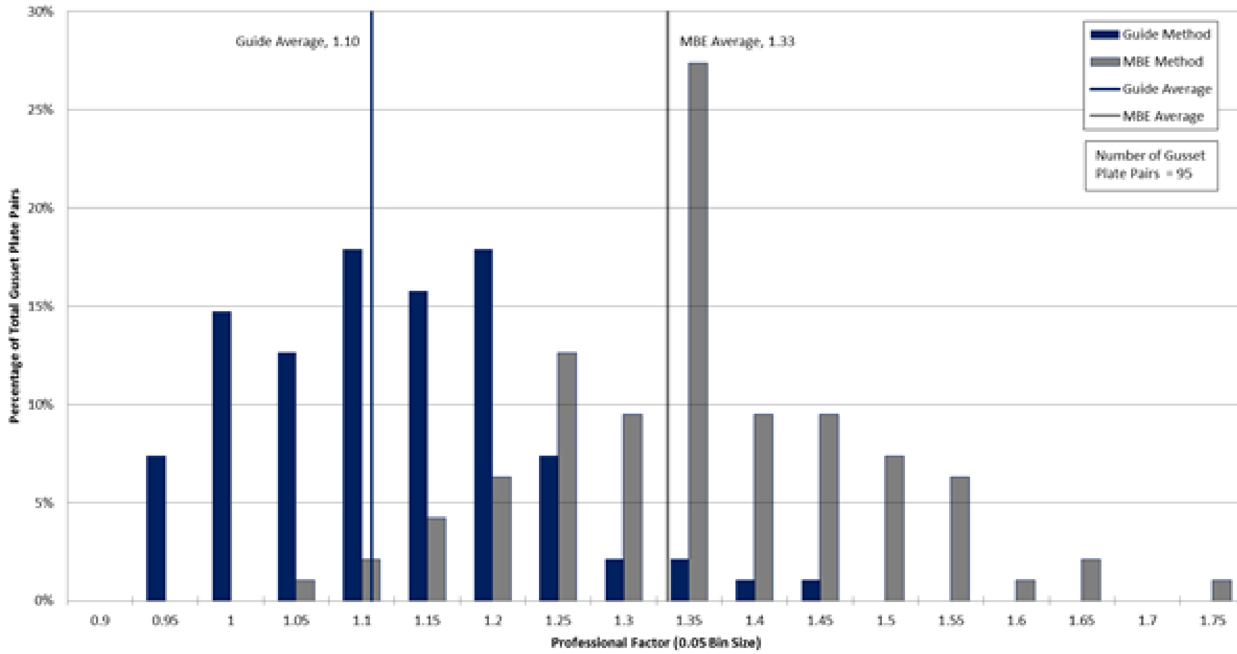


Figure 1. Comparison of professional factors between Guide and MBE.

Reliability

When using an approach other than the formulas contained in the Specs and the MBE to quantify gusset plate capacity, consideration must be given to the notion of structural reliability. At the very least, an alternate method must be based on rational engineering principles and provide reasonable estimates of actual plate strengths. The tricky part of this requirement is defining "reasonable." One definition would be something that provides a level of reliability comparable to that inherent in the original design procedure. This is the essence of "grandfathering," a concept embraced by building codes as long as building codes have been updated. Without grandfathering, structures would have to be retrofit every time codes and standards become more stringent.

While grandfathering is a reasonable—even necessary—concept to apply to the evaluation of existing structures, there are instances where it may fall short of achieving the desired ends of providing and maintaining reasonable reliability. For example, if the original design method included a substantial technical error or omission that resulted in unacceptably low levels of reliability, evaluation of the resulting structures should be based on a modified approach. In the context of major truss gusset plates, an argument can be made that historic design methods were less than comprehensive; that in some cases, potential failure modes were overlooked to the extent that plate capacity and reliability is less than what was intended, or less than what is currently desired. At the same time, it is clearly unreasonable to expect existing, often decades old, plates to meet current standards for new construction. For example, if common design practices in the 1940s routinely provided relative reliability factors (β), discussed in detail in later sections, in the 2.0 to 3.0 range, it might be unreasonable to require these structures to now be substantially more reliable simply because we want new structures to achieve higher β values. At the same time, if an older practice allowed certain types of plates to be built with β values well below the values provided by most or all other elements of the structure, we might want to evaluate these plates using higher standards. To our knowledge, no studies have been done to evaluate this issue.

The basic MBE formulas are based on achieving a minimum β value of 3.5. In fact, a benchmark β of 3.5 was used to determine strength reduction factors (ϕ factors) for the various strength formulas. Since the MBE ϕ factors are based on achieving a β of 3.5, an argument can be made that using them to evaluate older structures—structures that were never expected to provide such reliability—is excessive.

Another issue involving ϕ factors relates to the use of alternate strength determination methods. Since the listed ϕ factors were calibrated to provide a specific level of reliability using the standard formulas, use of a different method may require different ϕ factors if the same level of reliability is to be established.

When used as part of a LRFD-based evaluation, the Guide methods will provide levels of reliability comparable to what would be provided by following the MBE approach based on excellent FE models. They will also provide levels of reliability comparable to, if not better than, that which was inherent in historical design practices. A detailed discussion of relative reliability in the context of the MBE and Guide methods is provided in Appendix C.

Ductility

Structural steel has considerable ductility. This means it can sustain strains well beyond yield level without degrading. Many current AASHTO and AISC capacity formulations rely on the material being ductile. If this were not the case, residual stresses, deformation-induced (e.g., fabrication-related) stresses, and forces and stresses caused by bolt pretension and weld shrinkage would have to be considered in many formulas that currently ignore them. Even the strength formulas for initially stress-free compact flexural members, shear plates, bolt groups, weld groups, and other elements would need to be greatly modified.

The Guide capacity calculation methods take advantage of the ductility of structural steel elements in ways that are consistent with AASHTO and AISC steel design standards.

Buckling Considerations

While structural steel is a relatively ductile material, there are circumstances under which some form of instability limits the ability of an element to sustain post-yield strains, or even precludes achievement of yield level strains. When an element, or the system of which the element is a part, becomes unstable before yield level strains are reached, the instability is often referred to as "elastic buckling." When instability follows achievement of yield level strains, it is often referred to as "inelastic buckling."

Sections of gusset plates that are not stiffened by connected members or other means can be susceptible to both elastic and inelastic buckling. Therefore, it is important for any capacity determination methods to account for these potential failure modes. When a connection detail includes unstiffened sections of plate surrounding the end of a web compression member, the Guide methods evaluate the potential for buckling and, if appropriate, reduce the maximum stresses that can be sustained in affected areas.

The Guide compression strength calculation methods are based on "Corner Checks." For a particular compression web member in an un-deteriorated plate, the "corner" is the section of gusset plate that contains the web member/plate fasteners, and is bounded by a vertical line that passes through the fastener (or row of fasteners) that is closest to the adjacent web member, and a horizontal line that passes through the fastener (or row of fasteners) that is closest to the chord. The section of plate defined by the vertical line is called the "vertical surface" of the corner, while the section of plate defined by the horizontal line is called the "horizontal surface" of the corner. A typical "corner" and associated surfaces are shown in Appendix B.

For each corner surface, there is a section of plate that spans between it and the nearest truss member. Examples are shown in Appendix B. In gusset plates that lack any external stiffening across these spans, buckling of the spans characterized by lateral movement of the compression member end relative to the other truss members can occur. This mode of instability is called "sidesway buckling." The shorter of the horizontal and vertical spans determines the strength of the sidesway buckling mode. The Guide approach treats the shorter span like a fixed-ended, sidesway column, and calculates a corresponding critical stress. This critical stress is used as an upper bound for the principal stress that the material making up the span can sustain. The longer span is treated like a bounded plate element. The loaded edges of the plate are assumed to be restrained against translation and rotation, one of the remaining sides is also assumed to be restrained against translation and rotation, and the fourth side is free of all restraint. The classic plate buckling equation is used to determine the critical Euler (i.e., elastic) buckling stress, while the actual critical buckling stress is determined using the AASHTO column buckling equation in order to account for the presence of residual stresses. As with the short span, the critical plate buckling stress defines the upper limit to the principal stresses in the longer span.

When the principal stress in a span falls below the corresponding critical stress, buckling of that span is not a factor. When the principal stress exceeds the associated critical stress, the forces acting on the corresponding corner surface are reduced proportionally.

Horizontal Shear Check

Shear failure along a plane parallel to the chord often determines the ultimate strength of a gusset plate. This failure mode is called "Horizontal Shear" in the MBE and this Guide. Because the critical shear plane usually carries more than just shear force, the effect of these other forces on the shear capacity must be considered. The MBE accounts for this by reducing the available shear strength by a factor designated by the symbol Ω. In all cases, the MBE assumes $\Omega = 0.88$.

The Guide method of calculating Horizontal Shear strength is identical to the method used in the MBE, except for the fact that Ω is variable. In the Guide approach for connections that are not located at truss support bearings, Ω is calculated using a shear/moment interaction equation developed by Drucker (Drucker, 1956). The moment that coincides with the shear is equal to the horizontal shear force multiplied by the distance between the chord centerline (i.e., where moment is zero) and the horizontal plane in question. Since the moment is a function of the shear and the shear capacity is a function of the moment, calculating Ω is an iterative process. When a truss node coincides with a truss bearing, there are additional normal stresses acting on the horizontal shear plane. To account for these stresses, shear strength is determined based on the approach outlined in Appendix A.

The steps involved in calculating the shear strength of a horizontal shear plane for a connection that is not located at a truss support point are provided below. Parameters are defined in the Glossary of Terms section.

1. Locate critical Horizontal Shear (HS) plane using the following constraints:
   a. between chord and web members
   b. parallel to chord

   c. as far from chord as possible, without intersecting any web member fasteners (note: if elements of web members cross all planes that meet Constraints (a) and (b), Horizontal Shear Capacity (HSC) can be modified to account for the added HS capacity provided by the crossing elements)

2. Determine length of critical HS plane (L_{HS})
3. Determine eccentricity of critical HS plane (e_{HS})
4. Calculate plastic shear strength (V_P) and plastic moment strength (M_P) of the critical plane as follows:
   a. $V_P = 0.58(F_y)(L_{HS})t$
   b. $M_P = F_y[t(L_{HS})^2/4]$
5. At the nominal shear strength (V_N), the moment acting on the critical plane (M_{HS}) is:
   a. $M_{HS} = V_N(e_{HS})$
6. The nominal strength given M_{HS} is calculated as follows:
   a. $V_N = V_P\Omega = V_P[1-(M_{HS}/M_P)]^{0.25}$ or; $V_N = V_P[1-(V_N(e_{HS})/M_P)]^{0.25}$
7. Moving the V_P term to the left side of the equations in Item 6a shows that the Guide definition of Ω is as follows:
   a. $\Omega = V_N/V_P = [1-(M_{HS}/M_P)]^{0.25} = [1-(V_N(e_{HS})/M_P)]^{0.25}$

Since V_N is on both sides of the above equations, solving for it is an iterative process. Pick a value, plug it into the equation, and vary the value until both sides are equal. Starting with a value of $0.9V_P$ should lead to a rapid solution.

When the connection in question is located at a truss support point, the critical horizontal shear plane carries more than just moment and shear. It also carries large net normal forces between the truss web members and the bearing. In this case, the stresses associated with the net normal forces must be combined with the normal stresses caused by bending in order to calculate the associated shear strength using the method outlined in Appendix A.

Basic Corner Check (BCC)

The BCC involves calculation of the capacity of a "Corner" section. A "Corner" section is the section of gusset plate that contains the web member/plate fasteners, and is bounded by a line that passes through the fastener (or row of fasteners) that is closest to and parallel to the adjacent chord member, and an orthogonal line that passes through the fastener (or row of fasteners) that is closest to the adjacent web member. The line that parallels the chord is called the "horizontal surface" of the Corner, while the orthogonal line is called the "vertical surface" of the Corner. A typical "Corner" and associated surfaces are shown in Figure 2.

The maximum forces that can develop on the horizontal and vertical surfaces of a Corner are calculated based on the following constraints:

- Each surface carries a uniform normal stress (σ) and a uniform shear stress (τ)
- σ and τ are limited by the Von Mises stress interaction:
  - $\sigma^2 + 3\tau^2 \leq F_y^2$
- The resultant of the shear and normal forces acting on each surface must pass through the connection work point

- The principal stress on a surface can be no more than the corresponding critical buckling stress calculated as previously described

The steps in implementing a BCC are outlined below:

1) Locate critical corner sides using the following constraints:
    a) horizontal side is parallel to chord, and passes through the compression member fastener(s) closest to the chord
    b) vertical side is orthogonal to the horizontal side, and passes through the compression member fastener(s) closest to the other web members
2) Locate the following points:
    a) centroid of vertical side (WP_v)
    b) centroid of horizontal side (WP_h)
    c) point where compression member centerline crosses the endmost row of compression member fasteners (WP_{S1})
    d) point where compression member centerline crosses the end of the compression member (WP_{S2})
    e) intersection of web and chord member centerlines (WP)
3) Calculate the following parameters:
    a) length of vertical side (L_v)
    b) length of horizontal side (L_h)
    c) angle between line drawn from WP to WP_V and vertical side (θ_v)
    d) angle between line drawn from WP to WP_H and horizontal side (θ_h)
    e) the longest of the following two distances (a):
        i) distance from WP_v to line through nearest fastener in adjacent web member (measured perpendicular to the vertical side)
        ii) distance from WP_h to line through nearest fastener in adjacent chord member (measured perpendicular to the horizontal side)
    f) distance between WP_{S1} and nearest edge of an adjacent web or chord member (L_{S1}) parallel to shorter distance “a”
    g) distance between WP_{S2} and nearest fastener in an adjacent web or chord member (L_{S2}) parallel to shorter distance “a”
4) Calculate the side forces (P_v, V_v, P_h, V_h) subject to the following constraints:
    a) $\sigma_{vm} = F_y$ on one of the sides, while $\sigma_{vm} \leq F_y$ on the other
    b) resultant of each set of side forces passes through WP (i.e., $P_v/V_v = \tan(\theta_v)$, and $P_h/V_h = \tan(\theta_h)$)
    c) resultant of all side forces aligns with compression member (i.e., $(P_h+V_v)/(P_v+V_h) = \tan(\theta_M)$
5) Calculate C_{BCC} as follows:
    a) $C_{BCC} = 2[(P_h+V_v)^2+(P_v+V_h)^2]^{0.5}$

Check Buckling

1) Calculate normal stress (σ) and shear stress (τ) on each side as follows:

   a) $\sigma_v = P_v/(L_v*t)$
   b) $\tau_v = V_v/(L_v*t)$
   c) $\sigma_h = P_h/(L_h*t)$
   d) $\tau_h = V_h/(L_h*t)$
2) Calculate principal stress (σ_{Princ}) on each side as follows:
   a) $\sigma_{Princ.v} = (\sigma_v/2) + [(\sigma_v/2)^2 + (\tau_v)^2]^{0.5}$
   b) $\sigma_{Princ.h} = (\sigma_h/2) + [(\sigma_h/2)^2 + (\tau_h)^2]^{0.5}$
3) Calculate critical buckling stress on side defined by L_{S1} and L_{S2} as follows:
   a) calculate $L_{Savg} = (L_{S1} + L_{S2})/2$
   b) calculate $KL/r = 1.0(L_{Savg})/(0.29t)$
   c) calculate Euler stress $F_e = \pi^2E/(KL/r)^2$
   d) calculate critical stress (F_{cr}) as follows:
      i) for $F_e > F_y/2$: $F_{cr} = F_y[1 - (F_y/F_e)^{0.5}/(2*2^{0.5})]$
      ii) for $F_e < F_y/2$: $F_{cr} = F_e$
4) Calculate critical buckling stress on side defined by “a” as follows:
   a) calculate the length/width ratio = a/b; (where b is either L_v or L_h, whichever is the one from which “a” was measured)
   b) calculate the buckling coefficient (k): $k = 4.64(a/b)^{-1.106}$
   c) calculate the critical elastic stress (F_e): $F_e = [k\,(\pi)^2\,E]/[10.6(b/t)^2]$; (where b is either L_V or L_H, whichever is the one from which “a” was measured)
   d) calculate critical stress (F_{cr}) as follows:
      i) for $F_e > F_y/2$: $F_{cr} = F_y[1 - (F_y/F_e)^{0.5}/(2*2^{0.5})]$
      ii) for $F_e < F_y/2$: $F_{cr} = F_e$
5) On each side, compare σ_{Princ} to corresponding F_{cr} and calculate P_C as follows:
   a) if $\sigma_{Princ} \leq F_{cr}$ on both sides; buckling is not a factor; $C_C = C_{BCC}$ from Step 5 of BCC check
   b) if $\sigma_{Princ} > F_{cr}$ on either or both sides; buckling is a factor; $C_C = [C_{BCC}$ from Step 5 of BCC check] x [smallest value of F_{cr}/σ_{Princ}]

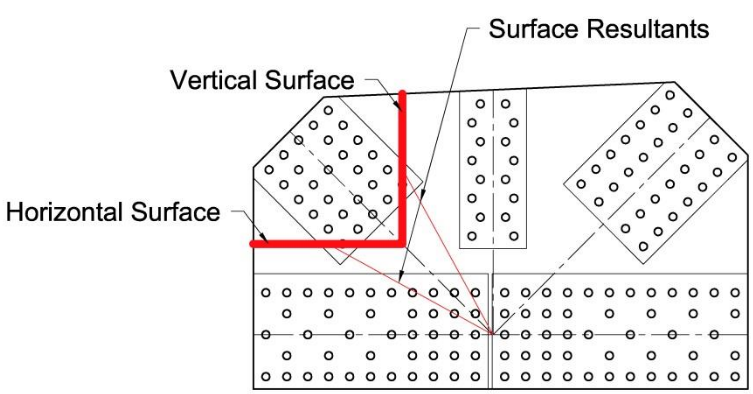


Figure 2. Basic Corner Check Surfaces and Resultants thru Work Point.

Refined Corner Check

This step involves an iterative process in which the BCC constraint that the resultant of each set of Corner side forces must pass through the WP is removed. This allows for side forces to make more effective use of available material strength. However, this also places greater demands on the portion of the gusset plate that is outside (essentially horizontally adjacent to) the compression Corner. As a result, that portion of plate must be checked for stability under the modified demands.

A typically effective starting point for the RCC is to assume the resultants of each set of side forces align with the axis of the compression member. If this proves to be a stable situation (i.e., if all remaining constraints are met, material stresses are not excessive, and buckling stresses are not exceeded), further iterations will likely yield very little increases in capacity. If the resulting side forces are not sustainable for any reason, the forces must be adjusted (usually by reducing the P force on one side and calculating the remaining 3 side forces), and the various checks re-done.

In all cases, the following constraints are maintained:

- The resultant of all 4 side forces aligns with compression member
- The sum of moments about the WP of all 4 side forces equals zero

When a RCC is done, the associated C_C supersedes the value calculated via the BCC. The steps involved with a RCC are outlined below:

1) Calculate initial P and V forces using the following constraints:
    a) select a specific C value > C_{BCC} for one of the sides (or, pick P/V ratio for a side that is greater than the ratio corresponding to the BCC case) and calculate the side forces subject to the following constraints:
    b) $\sigma_{vm} = F_y$ on one of the sides, while $\sigma_{vm} \leq F_y$ on the other
    b) resultant of side forces aligns with compression member (i.e., $(P_v + V_h)/(P_h + V_v) = \tan(\theta_M)$)
    c) sum of moments about WP of all P and V forces = zero
2) Calculate initial C_{RCC} as follows:
    a) $C_{RCC} = 2[(P_h+V_v)^2+(P_v+V_h)^2]^{0.5}$
3) Calculate forces in other web elements that act concurrently with C_{RCC}
4) Calculate axial load (P_Q) and moment (M_Q) acting on Q surface of “stub” of plate adjacent to compression corner
5) Calculate maximum normal stress acting on the Q surface as follows:
    a) $\sigma_{max} = P_Q/A_Q + M_Q/S_Q$
6) Calculate minimum normal stress acting on the Q surface as follows:
    a) $\sigma_{min} = P_Q/A_Q - M_Q/S_Q$
7) If $\sigma_{max} \leq F_y$, and σ_{min} is negative or equal to zero, calculate σ_i as follows; otherwise go to Step 8:
    a) $\sigma_i = 0.6(\sigma_{max})$; go to Step 9
8) If $\sigma_{max} \leq F_y$, and σ_{min} is positive, calculate σ_i as follows; otherwise go to Step 1 (σ_{max} is to high):
    a) $\sigma_i = \sigma_{min} + 0.6(\sigma_{max} - \sigma_{min})$; go to Step 9
9) Calculate available shear strength along the Q surface as follows:

   a) $\tau_N = 0.58(F_y)\,[1 - (\sigma_i/F_y)^2]^{0.5}$

10) If τ_N does not equal (or nearly equal) the sum of the horizontal stress acting on the Q surface , go to Step 1 (i.e., perform another iteration); otherwise, go to Step 11
11) Check principal compression Corner stresses for the RCC P_v, V_v, P_h, and V_h forces as outlined in the BCC check, then go Step 12
12) On each side, compare σ_{Princ} to corresponding F_{cr} and calculate C_C as follows:
    a) if $\sigma_{Princ} \le F_{cr}$ on both sides; $C_C = C_{RCC}$
    b) if $\sigma_{Princ} > F_{cr}$ on either or both sides; $C_C = C_{RCC}$ x [smallest value of F_{cr}/σ_{Princ}]

Deterioration

The MBE includes a method for accounting for the effects of certain types of plate deterioration. The method can be very conservative. When used to evaluate Whitmore L_{mid} buckling strengths, it tends to greatly overestimate the effect of localized deterioration by essentially projecting localized section losses along the entire length of the equivalent column.

The Corner Check methods outlined in this Guide provide a versatile way to more accurately assess the effects of deterioration, especially the common form of banded section loss that often exists along the top of the bottom chord member. The BCC and RCC checks can be supplemented by other Corner checks in which the Corner surfaces are located to intercept the zones of significant deterioration. In this way, the sections that define member capacity (in this case, both tension and compression), are reduced by the deterioration. In addition, variation in section loss along a surface is accounted for by locating the surface work point to match the center of gravity of the affected section. The effect of deterioration on buckling is accommodated by developing equivalent thicknesses for the plate material comprising each span.

The Guide also accounts for strain hardening in narrow bands of deterioration using the same approach relied upon by AASHTO and AISC to account for strain hardening in net sections. This avoids the overly conservative assumption that the maximum stress that can be mobilized at the root of a narrow strip of section loss is limited to F_Y.

In general, the effects of deterioration are evaluated by doing a number of additional Corner checks, each one intercepting different areas of deterioration. The check providing the lowest capacity governs. Since deterioration mechanisms act without the constraints of any codes or standards to create an infinite variety of conditions, it is impossible to provide quantitative methods (e.g., formulas) to address all forms of deterioration. However, concepts for accounting for the effects of deterioration can be provided.

Accurate assessment of deterioration requires accurate data. When considering various potential failure planes, it is important to know with some precision, the thickness of the material along that plane. In order to identify possible critical planes, it is important to know how thickness varies in all directions. Spending an extra hour measuring a plate’s thickness in areas of section loss can mean the difference between spending tens of thousands of dollars and spending nothing.

One general concept that should be applied whenever possible is strain hardening. The AASHTO and AISC design standards take advantage of strain hardening by allowing the use of the material’s ultimate strength in situations where the associated strains can be mobilized before failure occurs. The most common examples include welds, bolts and net sections (i.e., potential failure planes that intercept bolt holes and/or other local reductions in cross sectional areas). When deterioration is highly localized, it is often appropriate, and highly beneficial, to treat potential failure planes passing through the deteriorated area as net sections; to base the strength on F_U rather than F_Y.

A very common form of deterioration involves a narrow band of section loss located just above and parallel to the top of the bottom chord. The reduced cross sectional areas created by such bands are essentially the same as the reduced areas (net areas) created by a row of bolt or rivet holes. Therefore, it is reasonable to calculate the strength of a plane through the band using F_U rather than F_Y. Even in wide bands of section loss, strain hardening will occur if the areas of maximum loss are confined to narrow strips within the overall deteriorated zone. Taking advantage of strain hardening in such cases requires careful quantification of section loss in all directions.

[1]Drucker, D., *The Effect of Shear on the Plastic Bending of Beams*, American Society of Mechanical Engineers, NAMD Conference, Urbana, Illinois, June 1956

GLOSSARY OF TERMS

A_g	=	gross area of the plate resisting shear (in.^{2})
A_h	=	area of horizontal surface (in.^{2})
A_n	=	net area of the plate resisting shear (in.^{2})
A_v	=	area of vertical surface (in.^{2})
a	=	long span unbraced plate buckling length (in.)
a_h	=	long span unbraced plate buckling length for horizontal interface (in.)
a_v	=	long span unbraced plate buckling length for vertical interface (in.)
b	=	long span plate buckling surface length (in.)
C_{BCC}	=	basic corner check capacity (kip)
$C_{BCC.vM}$	=	basic corner check capacity based on von Mises stress on interfaces only, does not consider buckling (kip)
C_{HS}	=	horizontal shear capacity of plate (kip)
$C_{HS.M2}$	=	Member 2 capacity based on horizontal shear (kip)
C_{RCC}	=	refined corner check capacity (kip)
C_U	=	net shear rupture capacity of plate (kip)
C_Y	=	gross shear yielding capacity of plate (kip)
c_n	=	distance from elastic neutral axis to extreme fiber in bending based on net section properties (in.)
DL_{Mi}	=	unfactored member dead load (i=1, 2, 3, 4, 5) (kip)
d_h	=	fastener hole diameter (in.)
E	=	modulus of elasticity of steel (ksi)
e_{brg}	=	eccentricity from centroid of horizontal shear yield plane to intersection of plane and line of action of bearing (in.)
$e_{brg.U}$	=	eccentricity from centroid of horizontal shear rupture plane to intersection of plane and line of action of bearing (in.)
e_{HS}	=	eccentricity of horizontal shear plane (in.)
e_{Mi}	=	eccentricity from center of gravity of gusset plate stub to intersection of stub surface and line of action of member (i = 3, 4) (in.), eccentricity from centroid of horizontal shear yield plane to intersection of plane and line of action of member (i = 1, 2) (in.)
$e_{Mi.U}$	=	eccentricity from centroid of horizontal shear rupture plane to intersection of plane and line of action of member (i = 1, 2) (in.)
$e_{h.wp}$	=	eccentricity of horizontal surface from work point (in.)
$e_{v.wp}$	=	eccentricity of vertical surface from work point (in.)
$e_{Q.wp}$	=	eccentricity of gusset plate “stub” surface from work point (in.)
F_{cr}	=	critical buckling stress (ksi)
F_e	=	Euler buckling stress (ksi)
F_{Mi}	=	force in member based on a corresponding horizontal shear yield capacity (i = 1, 2, 3) (kip)
$F_{Mi.U}$	=	force in member based on a corresponding horizontal shear rupture capacity (i = 1, 2, 3) (kip)
$F_{RCC.Mi}$	=	equivalent concurrent forces in members (i = 3, 4) based on refined corner check capacity (kip)
F_u	=	specified minimum tensile strength of steel (ksi)
F_y	=	specified minimum yield strength of steel (ksi)
I_g	=	moment of inertia based on gross section properties (in.^{4})
I_n	=	moment of inertia based on net section properties (in.^{4})
$InvForce_{Mi}$	=	factored member forces for inventory rating (i = 1, 2, 3, 4, 5) (kip)
IRF_{BCC}	=	inventory rating factor based on basic corner check capacity
IRF_{HS}	=	inventory rating factor based on horizontal shear capacity
IRF_{RCC}	=	inventory rating factor based on refined corner check capacity

K	=	effective length factor
k	=	plate buckling coefficient
L	=	length of controlling partial shear plane (in.); length of full shear plane (in.)
L_c	=	distance from the middle of the Whitmore section to the nearest member fastener line in the direction of the member (in.)
L_h	=	length of horizontal surface (in.)
L_Q	=	horizontal length of gusset plate outside of corner (length of “stub”) (in.)
L_s	=	average of unbraced lengths for column buckling (in.)
L_{s1}	=	unbraced length for column buckling measured orthogonally to surface with smaller of the unbraced plate buckling lengths. Distance is from the intersection of member centerline with the row of rivets nearest work point to nearest member edge (in.)
L_{s2}	=	unbraced length for column buckling measured orthogonally to surface with smaller of the unbraced plate buckling lengths. Distance is from the intersection of member centerline with the leading member edge to nearest fastener of another truss member (in.)
L_U	=	length of full shear plane for plate rupture (in.)
L_v	=	length of vertical surface (in.)
L_Y	=	length of full shear plane for plate yielding (in.)
LL_{Mi}	=	unfactored member live load (i=1, 2, 3, 4, 5) (kip)
M	=	moment demand along shear yield plane (k-in.)
M_P	=	plastic moment capacity (k-in.)
M_{Plane}	=	moment on horizontal shear yield plane (k-in.)
$M_{Plane.U}$	=	moment on horizontal shear rupture plane (k-in.)
M_Q	=	moment on gusset plate “stub” (k-in.)
n_{hole}	=	number of fastener holes in net section
$OpForce_{Mi}$	=	factored member forces for operating rating (i = 1, 2, 3, 4, 5) (kip)
ORF_{BCC}	=	operating rating factor based on basic corner check capacity
ORF_{HS}	=	operating rating factor based on horizontal shear capacity
ORF_{RCC}	=	operating rating factor based on refined corner check capacity
P_h	=	axial component of resultant on horizontal surface (kip)
P_{Plane}	=	axial force on horizontal shear yield plane (kip)
$P_{Plane.U}$	=	axial force on horizontal shear rupture plane (kip)
P_Q	=	axial force on gusset plate “stub” (kip)
P_v	=	axial component of resultant on vertical surface (kip)
Ratio	=	available stress to demand stress
R_{brg}	=	bearing reaction based on horizontal shear yield capacity (kip)
$R_{brg.U}$	=	bearing reaction based on horizontal shear rupture capacity (kip)
r	=	radius of gyration (in.)
S_g	=	section modulus based on gross section properties (in.^{3})
S_n	=	section modulus based on net section properties (in.^{3})
S_Q	=	section modulus of gusset plate “stub”(in.^{3}) mention subscripts n,g
t	=	plate thickness (in.)
V	=	shear demand along shear yield plane (kip)
V_h	=	shear component of resultant on horizontal surface (kip)
V_P	=	plastic shear capacity (kip)
V_{Plane}	=	shear force on horizontal shear yield plane (kip)
$V_{Plane.U}$	=	shear force on horizontal shear rupture plane (kip)
V_Q	=	shear force on gusset plate stub (kip)
V_v	=	shear component of resultant on vertical surface (kip)

v_Q = actual shear stress on gusset plate "stub" (ksi)
W_{whit} = width of Whitmore section (in.)
$y_{bar.left}$ = distance to centroid of gusset plate surface from the left edge (in.)
$y_{bar.right}$ = distance to centroid of gusset plate surface from right edge (in.)
$y_{bar.v}$ = distance to centroid of gusset vertical surface of corner check the bottom edge (in.)
y_{PNAr} = distance to plastic neutral axis from right edge of gusset plate (in.)
$y_{bar.Q}$ = distance to centroid of gusset "stub" of the refined corner check from the left edge of "stub" (in.)
θ_h = angle between resultant thru work point on horizontal surface and horizontal surface (degrees)
θ_{Mi} = angle of member to chord (i = 1, 2, 3, 4, 5) (degrees)
$\theta_{PanelPoint}$ = angle of bottom chord with respect to the horizontal (degrees)
θ_v = angle between resultant thru work point on vertical surface and vertical surface (degrees)
ν = material Poisson's ratio
σ = normal stress (ksi)
σ_h = normal stress on horizontal surface (ksi)
σ_M = bending stress on gusset plate "stub" (ksi)
σ_P = axial stress on gusset plate "stub" (ksi)
$\sigma_{0.6}$ = equivalent normal stress to be used in von Mises relationship (ksi)
σ_{Princ} = principal stress (ksi)
σ_v = normal stress on vertical surface (ksi)
σ_{vm} = von Mises stress (ksi)
$\sigma_{vm.v}$ = von Mises stress on vertical surface (ksi)
$\sigma_{vm.h}$ = von Mises stress on horizontal surface (ksi)
τ = shear stress (ksi)
τ_h = shear stress on horizontal surface (ksi)
τ_N = maximum allowable shear stress on gusset plate "stub" (ksi)
τ_v = shear stress on vertical surface (ksi)
ϕ_{vu} = resistance factor for gusset plate shear rupture taken as 0.85
ϕ_{vy} = resistance factor for gusset plate shear yielding taken as 1.00
Ω = shear reduction factor for gusset plates

Example 1 - Noncompact Gusset Plate with Short Vertical Buckling Length

Load Factor Rating (LFR) Method

Example 1 is a five member gusset plate with a short buckling length between members. It is not a compact gusset plate and no members are chamfered. Calculations apply to one of two gusset plates.

G1.1 Gusset Plate Material, Geometric, and Loading Properties:

Material Properties

$F_y = 36.4\text{ksi}$

$F_u = 62.6\text{ksi}$

$E = 29000\text{ksi}$

$\nu = 0.3$

Plate Thickness

$t = \frac{1}{2}\text{in}$

Member Angles

$\theta_{M2} = 45\text{deg}$

$\theta_{M3} = 90\text{deg}$

$\theta_{M4} = 45\text{deg}$

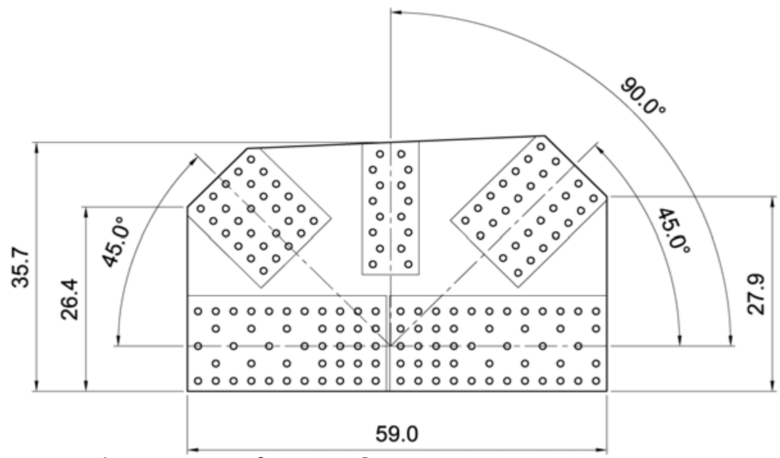


Figure 1: Basic Geometry of Gusset Plate

Unfactored Member Forces Per Gusset Plate Pair

$LL_{M1} = 80\,\text{kip}$

$DL_{M1} = 186\,\text{kip}$

$LL_{M2} = -165\,\text{kip}$

$DL_{M2} = -386\,\text{kip}$

$LL_{M3} = 33\,\text{kip}$

$DL_{M3} = 76\,\text{kip}$

$LL_{M4} = 117\,\text{kip}$

$DL_{M4} = 273\,\text{kip}$

$LL_{M5} = -120\,\text{kip}$

$DL_{M5} = -280\,\text{kip}$

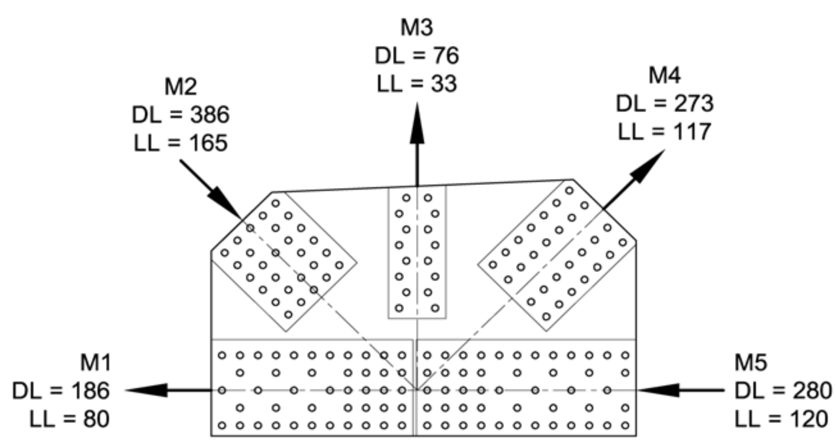


Figure 2: Concurrent Member Forces Transferred to Two Gusset Plates

Member forces based on NCHRP Project 12-84 loads with an assumed Dead Load to Live Load ratio of 70/30.

G1.1 Gusset Plate Material, Geometric, and Loading Properties Cont.:

Factored Forces Acting on Gusset Plate Pair

$InvForce_{M1} = \gamma_{InvLL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 2.17 \cdot 80kip + 1.3 \cdot 186kip$

$OpForce_{M1} = \gamma_{LL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 1.3 \cdot 80kip + 1.3 \cdot 186kip$

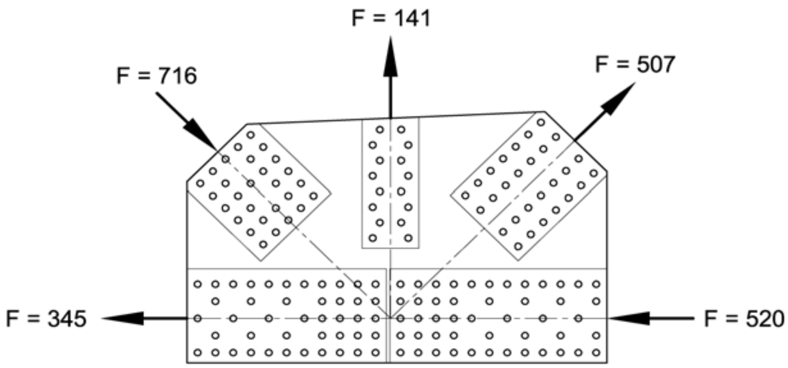


Figure 3: Concurrent Member Operating Forces Transferred to Two Gusset Plates

$InvForce_{M1} = 414\,kip$

$OpForce_{M1} = 345\,kip$

$InvForce_{M2} = -860\,kip$

$OpForce_{M2} = -716\,kip$

$InvForce_{M3} = 169\,kip$

$OpForce_{M3} = 141\,kip$

$InvForce_{M4} = 609\,kip$

$OpForce_{M4} = 507\,kip$

$InvForce_{M5} = -624\,kip$

$OpForce_{M5} = -520\,kip$

G1.2 Evaluation Approach:

In accordance with the *2014 Interim Revisions to the Manual for Bridge Evaluation, Second Edition,* the following gusset plate limit state checks were done:
(a) Fastener strength (L6B.2.6.1)
(b) Vertical shear resistance (L6B.2.6.3)
(c) Horizontal shear resistance (L6B.2.6.3)
(d) Partial shear yield resistance (L6B.2.6.3)
(e) Compressive (Whitmore) resistance (L6B.2.6.4)
(f) Tension strength (L6B.2.6.5)
(g) Bock shear resistance (L6B.2.6.5)
(h) Chord splice capacity (L6B.2.6.6)

Load Factor Rating Summary for Example 1

Limit State	Gusset Plate Pair	
	Operating Rating	Inventory Rating
Fasteners	4.42	2.65
Vertical Shear	4.08	2.44
Horizontal Shear	1.89	1.13
Partial Shear Yield	0.61	0.37
Whitmore Compression	0.95	0.57
Tension	4.32	2.59
Block Shear	4.18	2.51
Chord Splice	16.1	9.64

7/8 in. diam A325 threads excluded fasteners

$\Omega = 0.88$ with splice plates included

Controls

When the Partial Shear Plane Yield and/or Whitmore Compression capacity checks control and indicate a less than acceptable rating, more rigorous evaluation should be performed.

The following more rigorous rating checks are performed in Example 1:
(1) Horizontal shear capacity - Ω calculated: Supercedes Horizontal Shear with $\Omega = 0.88$.
(2) Basic Corner Check capacity (BCC): Replaces Partial Shear Plane Yield and Whitmore Compression capacity che
(3) Refined Corner Check capacity (RCC): Supercedes BCC unless BCC indicates acceptable rating.

G1.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω):

Global shear check along horizontal plane parallel with bottom chord. Shear force calculated using horizontal component of diagonal member forces. Gross section selected at bottom fastener of diagonal and vertical members to achieve maximum eccentricity. Net section calculated through bottom chord fastener holes. Ω calculated using Drucker formula.

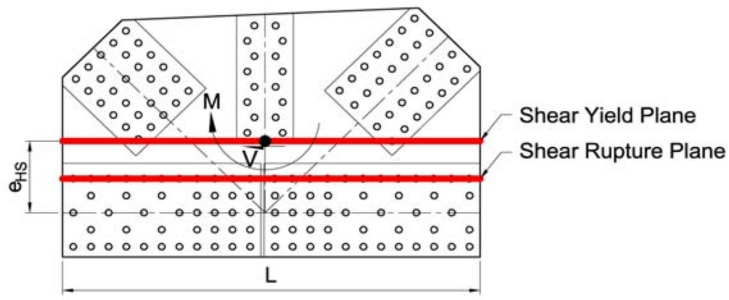


Figure 4: Horizontal Shear Between Web and Chord Members

$L = 59.0\ \text{in}$

$e_{HS} = 10.52\ \text{in}$

$M = V \cdot e_{HS}$

$A_g = t \cdot L_{HS} = 0.5\text{in} \cdot 59.0\text{in} = 29.5\text{in}^2$

$d_h = 1\ \text{in}$

$n_{hole} = 23$

$A_n = t \cdot \left(L - n_{hole} \cdot d_h\right) = 0.5\text{in} \cdot [59.0\text{in} - (23) \cdot 1.0\text{in}] = 18.0\text{in}^2$

Calculate Ω using Drucker formula instead of using AASHTO-specified Ω=0.88

$$V = V_p \cdot \left[1 - \left(\frac{M}{M_p}\right)\right]^{0.25} \qquad V = \Omega \cdot V_p$$

Drucker Formula [1]

$$V_P = (0.58) \cdot F_y \cdot A_g = (0.58) \cdot 36.4\text{ksi} \cdot 29.5\text{in}^2 = 623\text{kip}$$

$$M_P = \frac{L^2 \cdot t}{4} \cdot F_y = \frac{(59.0\text{in})^2 \cdot 0.5\text{in}}{4} \cdot 36.4\text{ksi} = 15800\text{in} \cdot \text{kip}$$

Substitute $V = \Omega * V_p$ into Drucker formula and rearrange to solve for Ω using plastic shear and moment capacities

$$\Omega \cdot V_p = V_p \cdot \left(1 - \frac{\Omega \cdot V_p \cdot e_{HS}}{M_p}\right)^{0.25}$$

$$\Omega = \left(1 - \frac{\Omega \cdot V_P \cdot e_{HS}}{M_P}\right)^{0.25} = \left(1 - \frac{\Omega \cdot 623\text{kip} \cdot 10.52\text{in}}{15800\text{in} \cdot \text{kip}}\right)^{0.25} = 0.89$$

Requires iterative process since V is proportional to Ω. Can substitute AASHTO specified value of Ω = 0.88 on right side of equation as a first estimate of Ω. Result shown is the calculated value of Ω after performing necessary iterations.

G1.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω) Cont.:

$\phi_{vy} = 1.0$

$\phi_{vu} = 0.85$

$C_Y = \phi_{vy} \cdot (0.58) \cdot F_y \cdot A_g \cdot \Omega = 1.00(0.58) \cdot 36.4\text{ksi} \cdot 29.5\text{in}^2 \cdot (0.89) = 555\text{kip}$

$C_U = \phi_{vu} \cdot (0.58) \cdot F_u \cdot A_n = 0.85(0.58) \cdot 62.6\text{ksi} \cdot 18.0\text{in}^2 = 556\text{kip}$

$C_{HS} = \min(C_Y, C_U) = \min(555\text{kip}, 556\text{kip}) = 555\text{kip}$

Horizontal Shear Capacity (per plate)

Determine capacity of member M2 based on Horizontal Shear

$$C_{HS.M2} = C_{HS} \cdot \left| \frac{OpForce_{M2}}{OpForce_{M1} - OpForce_{M5}} \right| = 555\text{kip} \cdot \left| \frac{-716\text{kip}}{345\text{kip} - (-520\text{kip})} \right| = 460\text{kip}$$

Total member capacity
$2 \cdot 460\text{kip} = 919\text{kip}$

$$ORF_{HS} = \frac{C_{HS.M2} - \gamma_{DL} \cdot \left| \frac{1}{2} DL_{M2} \right|}{\gamma_{LL} \cdot \left| \frac{1}{2} LL_{M2} \right|} = \frac{460\text{kip} - 1.3 \cdot \left| \frac{1}{2} \cdot -386\text{kip} \right|}{1.3 \cdot \left| \frac{1}{2} \cdot -165\text{kip} \right|} = 1.95$$

$$IRF_{HS} = \frac{C_{HS.M2} - \gamma_{DL} \cdot \left| \frac{1}{2} DL_{M2} \right|}{\gamma_{InvLL} \cdot \left| \frac{1}{2} LL_{M2} \right|} = \frac{460\text{kip} - 1.3 \cdot \left| \frac{1}{2} \cdot -386\text{kip} \right|}{2.17 \cdot \left| \frac{1}{2} \cdot -165\text{kip} \right|} = 1.16$$

[1] Drucker, D., *The Effect of Shear on the Plastic Bending of Beams*, American Society of Mechanical Engineers, NAMD Conference, Urbana, IL, June 1956

G1.2.2 Basic Corner Check:

The Basic Corner Check is a first-principles analytical approach utilizing fundamental steel design theory to conservatively calculate gusset plate limit state capacities at critical cross sections. This check is used to evaluate equilibrium and stability of a gusset plate "corner" bounded by horizontal and vertical planes that create the smallest section encompassing all fasteners of the diagonal member. The diagonal member force is assumed to be resisted by a combination of shear and normal forces acting on the vertical and horizontal surfaces bounding the "corner". Von Mises stress calculated on the surfaces is limited to the yield strength of the gusset plate. For simplicity and to avoid bending in the members, the resultant of each surface must pass through the work point. The "corner" can be adjusted in terms of location and plate thickness to accommodate deterioration.

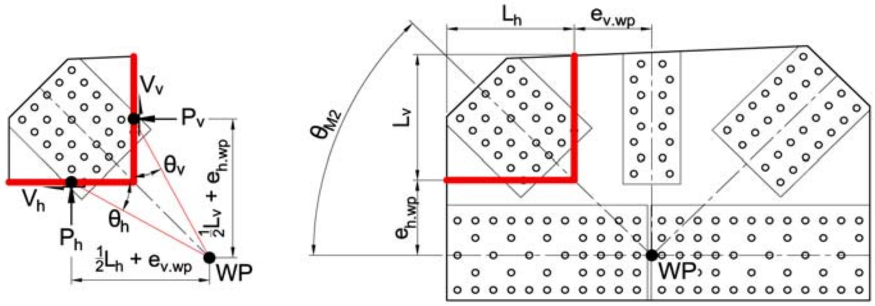


Figure 5: Basic Corner Check for Diagonal Member M2

Calculate resultant angles from the work point

$L_h = 17.8\,in$ $\qquad e_{h.wp} = 10.8\,in$

$L_v = 18\,in$ $\qquad e_{v.wp} = 10.8\,in$

$$\theta_h = atan\left(\frac{e_{h.wp}}{\frac{L_h}{2} + e_{v.wp}}\right) = atan\left(\frac{10.8in}{\frac{17.8in}{2} + 10.8in}\right) = 28.7deg$$

$$\theta_v = atan\left(\frac{e_{v.wp}}{\frac{L_v}{2} + e_{h.wp}}\right) = atan\left(\frac{10.8in}{\frac{18.0in}{2} + 10.8in}\right) = 28.6deg$$

G1.2.2a Horizontal Surface Check:

Since $L_h < L_v$ set von Mises stress on horizontal surface equal to plate yield strength. After stresses on both surfaces are determined, verify assumption that horizontal surface is critical (i.e. reaches von Mises yield before vertical surface).

$$P_h = V_h \cdot \tan(\theta_h)$$

$$\sigma_h = \frac{P_h}{A_h} = \frac{P_h}{L_h \cdot t}$$

$$\tau_h = \frac{V_h}{A_h} = \frac{V_h}{L_h \cdot t}$$

$$\sigma_{vm} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}}$$

Substitute P_h as a function of V_h and set the von Mises stress to yield

$$F_y = 36.4\text{ksi} = \sigma_{vm} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}} = \sqrt{\left(\frac{P_h}{L_h \cdot t}\right)^2 + 3 \cdot \left(\frac{V_h}{L_h \cdot t}\right)^2} = \sqrt{\left(\frac{V_h \cdot \tan(\theta_h)}{L_h \cdot t}\right)^2 + 3 \cdot \left(\frac{V_h}{L_h \cdot t}\right)^2}$$

Rearrange terms and solve for V_h

$$V_h = \frac{F_y \cdot L_h \cdot t}{\sqrt{\tan(\theta_h)^2 + 3}} = \frac{36.4\text{ksi} \cdot 7.8\text{in} \cdot 0.5\text{in}}{\sqrt{\tan(28.7\text{deg})^2 + 3}} = 178\text{kip}$$

Solve for P_h

$$P_h = V_h \cdot \tan(\theta_h) = 178\text{kip} \cdot \tan(28.7\text{deg}) = 98\text{kip}$$

Calculate shear and normal stresses on horizontal surface

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{98\text{kip}}{(17.8\text{in}) \cdot (0.5\text{in})} = 11.0\text{ksi} \qquad \tau_h = \frac{V_h}{L_h \cdot t} = \frac{178\text{kip}}{(17.8\text{in}) \cdot (0.5\text{in})} = 20.0\text{ksi}$$

G1.2.2b Vertical Surface Check:

Determine forces and stresses on vertical surface based on horizontal surface forces and stated constraints (i.e. force resultants to pass through workpoint).

$$P_v = V_v \cdot \tan(\theta_v)$$

$$\theta_v = 28.6 \cdot \text{deg}$$

Constrain final resultant to act along member

$$\theta_{M2} = \text{atan}\left(\frac{V_v + P_h}{P_v + V_h}\right)$$

G1.2.2b Vertical Surface Check Cont.:

Substitute P_v as a function of V_v

$$\theta_{M2} = \operatorname{atan}\left(\frac{V_v + P_h}{P_v + V_h}\right) = \operatorname{atan}\left(\frac{V_v + P_h}{V_v \cdot \tan(\theta_v) + V_h}\right)$$

Rearrange terms and solve for V_v. Substitute values obtained from previously solving P_h and V_h.

$$V_v = \frac{P_h - V_h \cdot \tan(\theta_{M2})}{\tan(\theta_{M2}) \cdot \tan(\theta_v) - 1} = \frac{98\text{kip} - 178\text{kip} \cdot \tan(45\text{deg})}{\tan(45\text{deg}) \cdot \tan(28.6\text{deg}) - 1} = 177\text{kip}$$

Solve for P_v

$$P_v = V_v \cdot \tan(\theta_v) = 178\text{kip} \cdot \tan(28.6\text{deg}) = 97\text{kip}$$

Calculate shear and normal stresses on vertical surface

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{97\text{kip}}{(18.0\text{in}) \cdot (0.5\text{in})} = 10.7\text{ksi} \qquad \tau_v = \frac{V_v}{L_v \cdot t} = \frac{177\text{kip}}{(18.0\text{in}) \cdot (0.5\text{in})} = 19.7\text{ksi}$$

Calculate von Mises stress

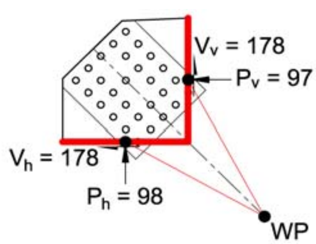
$$\sigma_{vm.v} = \sqrt{\sigma_v^{\,2} + 3\tau_v^{\,2}} = \sqrt{(10.7\text{ksi})^2 + 3 \cdot (19.7\text{ksi})^2} = 35.7\text{ksi} \le F_y = 36.4\,\text{ksi}$$

Since von Mises stress on vertical surface is less than yield strength of the gusset plate, the horizontal surface controls. If this had not been the case, the von Mises stress calculated on the vertical surface would have been greater than the yield stress. The previous process would have been modified by first setting the von Mises stress on the vertical surface to the yield stress and then determining the necessary resultants on the horizontal surface to balance the moment about the work point.

Substitute corresponding solved forces to determine member resultant force.

$$C_{BCCvM} = \sqrt{(V_h + P_v)^2 + (V_v + P_h)^2} = \sqrt{(178\text{kip} + 97\text{kip})^2 + (178\text{kip} + 98\text{kip})^2} = 389\text{kip}$$

BCC von Mises Capacity (per plate)



Total member capacity
$2 \cdot 389\text{kip} = 778\,\text{kip}$

Figure 6: Basic Corner Check Resultants for Diagonal Member M2

G1.2.2c BCC Buckling Check:

Check plate buckling due to axial forces on Basic Corner Check surfaces (refer to Appendix B). If buckling controls, then von Mises stresses must be adjusted.

Evaluate buckling capacity of plate regions defined by the short and long span between the Basic Corner Check surfaces and adjacent member connections.

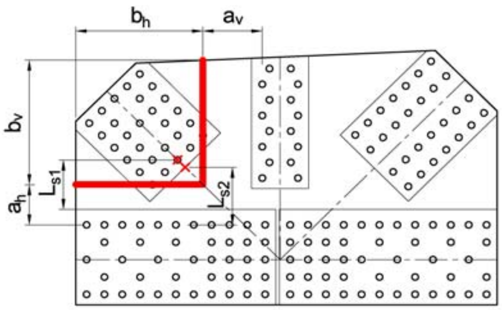


Figure 7: Corner Check Buckling Lengths

G1.2.2c1 Short Span Buckling Check:

For this gusset plate, the short span corresponds to the horizontal surface ($a_h < a_v$). a_h and a_v are defined as the distances from the respective Corner Check surface to the parallel line passing through the nearest fastener in an adjacent member.

$$L_s = \frac{L_{s1} + L_{s2}}{2} = \frac{7.1\text{in} + 8.3\text{in}}{2} = 7.7\text{in}$$

$$r = \frac{t}{\sqrt{12}} = \frac{0.5\text{in}}{\sqrt{12}} = 0.14\text{in}$$

Short span controls sidesway buckling, and rotation at each end is restrained. Therefore K = 1.0 used.

$$F_e = \frac{\pi^2 \cdot E}{\left(\frac{K \cdot L_s}{r}\right)^2} = \frac{\pi^2 \cdot 29000\text{ksi}}{\left(\frac{1.0 \cdot 7.7\text{in}}{0.14\text{in}}\right)^2} = 101\text{ksi}$$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2 \cdot \sqrt{2}}\right) = 36.4\text{ksi} \cdot \left(1 - \frac{\sqrt{\frac{36.4\text{ksi}}{101\text{ksi}}}}{2 \cdot \sqrt{2}}\right) = 28.7\text{ksi}$$

$$\sigma = \sigma_h = 11.0\text{ksi}$$

$$\tau = \tau_h = 20.0\text{ksi}$$

$$\sigma_{Princ} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{11.0\text{ksi}}{2} + \sqrt{\left(\frac{11.0\text{ksi}}{2}\right)^2 + (20.0\text{ksi})^2} = 26.3\text{ksi} \quad \le \quad F_{cr} = 28.7\text{ksi}$$

Principle stress is less than the critical buckling stress; therefore, buckling of short span does not control

G1.2.2c2 Long Span Buckling Check:

Treat long span as flat rectangular plate with one non-loaded edge fixed and the remaining edges clamped (dashed curve D in Figure 8)

Long Span Length (Figure 8)

$$a = a_v = 8.3\text{in}$$

Length of Long Side Surface (Figure 8)

$$b = L_v = 18.0\text{in}$$

$$\frac{a}{b} = 0.46$$

Because a/b is less than 0.75 (where k curve is nearly asymptotic), buckling of long span plate is not a concern. Otherwise calculate k as follows (using an approximate best fit function of dashed curve D in Figure 8):

$$k = 4.64 \cdot \left(\frac{a}{b}\right)^{-1.106}$$

$$F_e = \frac{k \cdot \pi^2 \cdot E}{12\left(1-\nu^2\right)\cdot\left(\frac{b}{t}\right)^2}$$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2\cdot\sqrt{2}}\right)$$

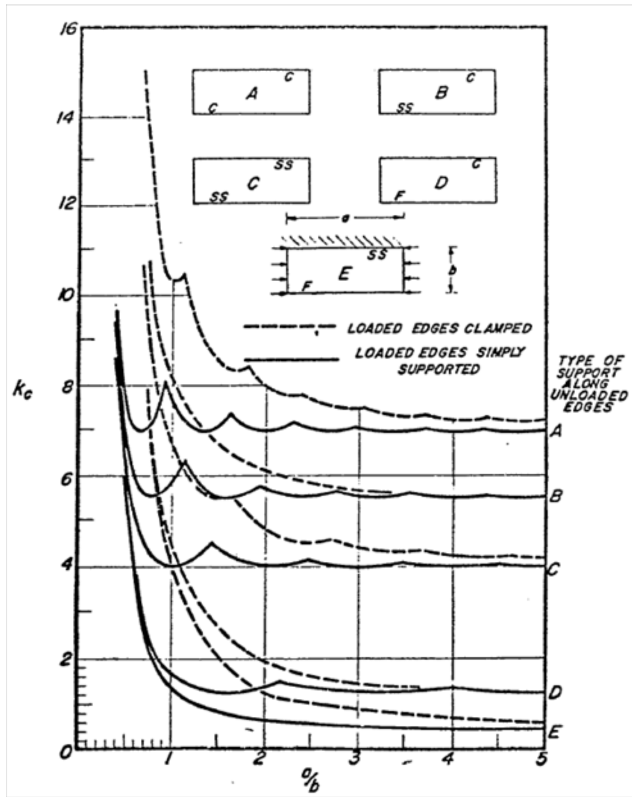


Figure 8: Elastic Buckling Coefficients [2]

Compare calculated principle stress to critical stress.

$$\sigma_{Princ} = \frac{\sigma_v}{2} + \sqrt{\left(\frac{\sigma_v}{2}\right)^2 + \tau_v^2} \quad \le \quad F_{cr}$$

G1.2.2 Basic Corner Check Cont.:

Since buckling of the short and long spans are not a concern for the Basic Corner Check, no reduction in calculated capacity is required and capacity calculated using von Mises stress applies.

$C_{BCC} = 389\,\text{kip}$

BCC Resultant Capacity (per plate)

Total member capacity

$2 \cdot 389\text{kip} = 778\,\text{kip}$

$$ORF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{389\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -386\text{kip}\right|}{1.3 \cdot |-165\text{kip}|} = 1.28$$

$$IRF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{389\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -386\text{kip}\right|}{2.17 \cdot |-165\text{kip}|} = 0.77$$

If an increased rating factor is required, perform a Refined Corner Check.

[2] George Gerard and Herbert Becker. *Handbook of Structural Stability*, Part I - Buckling of Flat Plates, Tech. Note 3871, National Advisory Committee for Aeronautics, Washington, D.C., July 1957.

G1.2.3 Refined Corner Check:

The Refined Corner Check removes the constraint that surface resultants pass through the work point as assumed in the Basic Corner Check. In removing this constraint, it is important to check the portion of gusset plate outside of the corner (Stub) and check again for plate buckling based on these resultants.

An efficient initial starting point in this iterative check is to force the resultants acting on each surface to be parallel to the member and then adjust shear and normal forces as necessary.

G1.2.3a Horizontal Surface Check: Parallel Resultants

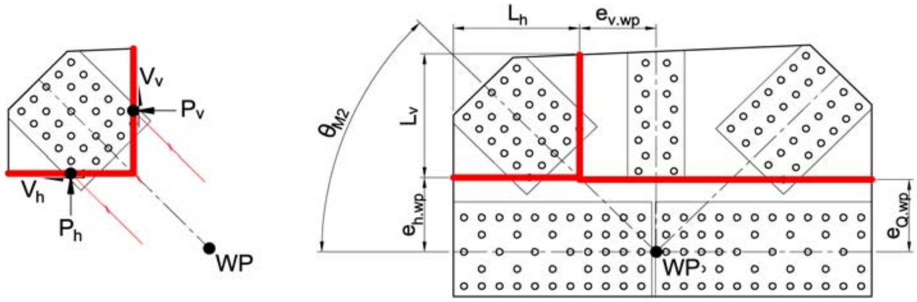


Figure 9: Refined Corner Check for Diagonal Member M2

As with the Basic Corner Check, check to see if the horizontal surface is the controlling surface by setting von Mises stress on horizontal surface equal to plate yield strength. After stresses on both surfaces are determined; verify assumption that horizontal surface is critical (i.e. reaches von Mises yield before vertical surface).

$$V_h = \frac{P_h}{\tan(\theta_{M2})}$$

Constrain von Mises stress on surface equal to the plate yield stress.

$$\sigma_{vm} = \sqrt{\sigma_h^2 + 3\tau_h^2} = F_y$$

$$L_h = 17.8\,in$$

$$\theta_{M2} = 45 \cdot deg$$

Substitute V_h as a function of P_h and set the von Mises stress to yield.

$$F_y = 36.4ksi = \sigma_{vm} = \sqrt{\sigma_h^2 + 3\tau_h^2} = \sqrt{\left(\frac{P_h}{L_h \cdot t}\right)^2 + 3\cdot\left(\frac{V_h}{L_h \cdot t}\right)^2} = \sqrt{\left(\frac{P_h}{L_h \cdot t}\right)^2 + 3\cdot\left(\frac{\frac{P_h}{\tan(\theta_{M2})}}{L_h \cdot t}\right)^2}$$

Rearrange terms and solve for P_h

$$P_h = \frac{F_y \cdot L_h \cdot t \cdot \tan(\theta_{M2})}{\sqrt{\tan(\theta_{M2})^2 + 3}} = \frac{36.4ksi \cdot 17.8in \cdot .5in \cdot \tan(45deg)}{\sqrt{\tan(45deg)^2 + 3}} = 162kip$$

Solve for V_h

$$V_h = \frac{P_h}{\tan(\theta_{M2})} = \frac{162kip}{\tan(45deg)} = 162kip$$

G1.2.3a Horizontal Surface Check Cont.: Parallel Resultants

Calculate resultants stresses on horizontal surface

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{162\text{kip}}{(17.8\text{in})\cdot(0.5\text{in})} = 18.2\text{ksi} \qquad \tau_h = \frac{V_h}{L_h \cdot t} = \frac{162\text{kip}}{(17.8\text{in})\cdot(0.5\text{in})} = 18.2\text{ksi}$$

G1.2.3b Vertical Surface Check: Parallel Resultants

Constrain moments about work point to balance (i.e. $\Sigma M_{WP} = 0$)

$$V_v = P_v \cdot \tan(\theta_{M2})$$

$$L_v = 18.0\text{ in}$$

$$e_{v.wp} = 10.8\text{ in}$$

$$e_{h.wp} = 10.8\text{ in}$$

$$\sum M = 0 = \left[P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) - V_h \cdot e_{h.wp}\right] + \left[-P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right) + V_v \cdot e_{v.wp}\right]$$

Substitute V_v as a function of P_v, rearrange terms and solve for P_v

$$0 = \left[P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) - V_h \cdot e_{h.wp}\right] + \left[-P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right) + P_v \cdot \tan(\theta_{M2}) \cdot e_{v.wp}\right]$$

$$P_v = \frac{\left[P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) - V_h \cdot e_{h.wp}\right]}{\left[\left(\frac{L_v}{2} + e_{h.wp}\right) - \tan(\theta_{M2}) \cdot e_{v.wp}\right]} = \frac{162\text{kip} \cdot \left(\frac{17.8\text{in}}{2} + 10.8\text{in}\right) - 162\text{kip} \cdot (10.8\text{in})}{\left(\frac{18\text{in}}{2} + 10.8\text{in}\right) - \tan(45\text{deg}) \cdot 10.8\text{in}} = 160\text{kip}$$

Solve for V_v

$$V_v = P_v \cdot \tan(\theta_{M2}) = 160\text{kip} \cdot \tan(45\text{deg}) = 160\text{kip}$$

Calculate resultants stresses on vertical surface

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{160\text{kip}}{(18.0\text{in})\cdot(0.5\text{in})} = 17.8\text{ksi} \qquad \tau_v = \frac{V_v}{L_v \cdot t} = \frac{160\text{kip}}{(18.0\text{in})\cdot(0.5\text{in})} = 17.8\text{ksi}$$

Calculate von Mises stress on vertical surface

$$\sigma_{vm.v} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{(17.8\text{ksi})^2 + 3\cdot(17.8\text{ksi})^2} = 35.5\text{ksi} \qquad \le \quad F_y = 36.4\text{ksi}$$

G1.2.3 Refined Corner Check Cont.: Parallel Resultants

Since the von Mises stress on the vertical surface is less than the yield strength of the gusset plate, the horizontal surface controls, as assumed. If this had not been the case, the von Mises stress calculated on the vertical surface would have been greater than the yield stress. The previous process would have been modified by first setting the von Mises stress on the vertical surface to the yield stress and then determining the necessary resultants on the horizontal surface to balance the moment about the work point.

$$C_{RCC} = \sqrt{\left(V_h + P_v\right)^2 + \left(V_v + P_h\right)^2} = \sqrt{(162\text{kip} + 160\text{kip})^2 + (160\text{kip} + 162\text{kip})^2} = 455\text{kip}$$

RCC Parallel Resultants Capacity (per plate)

Total member capacity
$2 \cdot 455\text{kip} = 910\,\text{kip}$

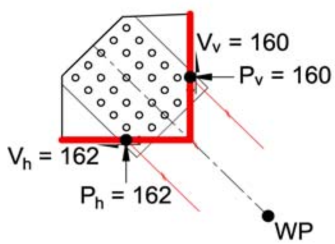


Figure 10: Refined Corner Check Resultants with Parallel Resultants to Member

G1.2.3c Remaining Portion (Stub) Check: Parallel Resultants

Determine equivalent concurrent forces for vertical and tension diagonal per plate

$$F_{RCC.M3} = OpForce_{M3} \cdot \left| \frac{C_{RCC}}{OpForce_{M2}} \right| = 141kip \cdot \left| \frac{455kip}{-716kip} \right| = 90kip$$

$$F_{RCC.M4} = OpForce_{M4} \cdot \left| \frac{C_{RCC}}{OpForce_{M2}} \right| = 507kip \cdot \left| \frac{455kip}{-716kip} \right| = 322kip$$

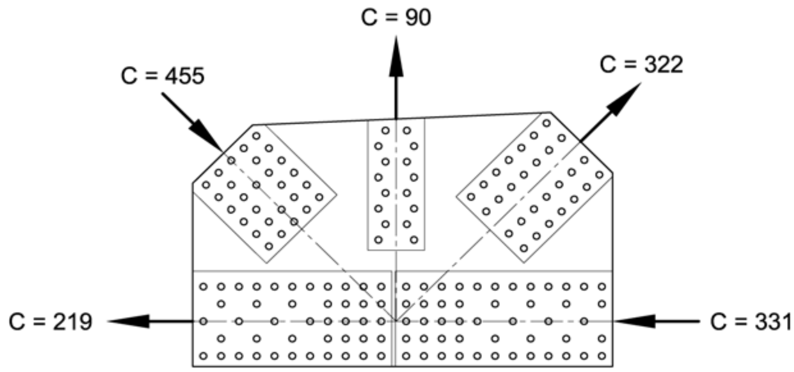


Figure 11: Concurrent Member Capacities (per plate) Based on Refined Corner Check (Subject to Stub Check and Buckling Check)

Check remaining portion of the gusset plate outside of the corner and chord. Select a Section Q that encompasses all forces applied by members M3 and M4.

$L_Q = 41.2in$

$e_{Q.wp} = 10.5\,in$

$\theta_{M4} = 45 \cdot deg$

$e_{M3} = 9.8\,in$

$e_{M4} = 0.7\,in$

$L_v = 18\,in$

$e_{h.wp} = 10.8\,in$

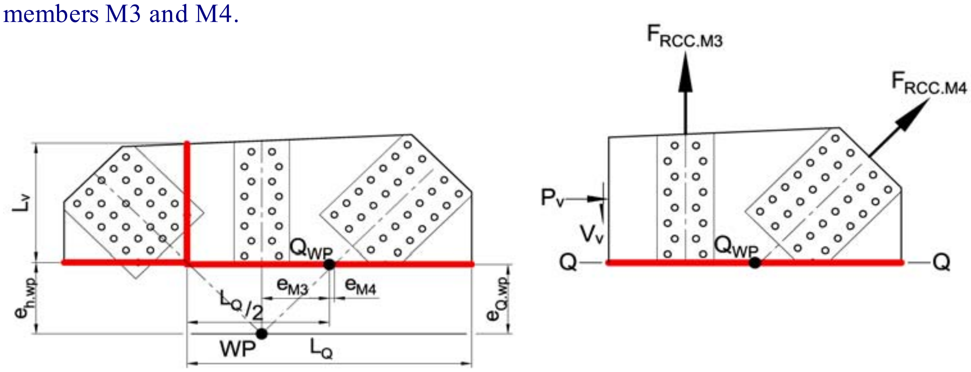


Figure 12: Remaining Gusset Plate Stub

G1.2.3c Remaining Portion (Stub) Check Cont.: Parallel Resultants

Calculate forces P_Q and V_Q along Section Q

$$P_Q = F_{RCC.M3} + F_{RCC.M4} \cdot \sin(\theta_{M4}) - V_v = 90kip + 322kip \cdot \sin(45deg) - 160kip = 158kip$$

$$V_Q = F_{RCC.M4} \cdot \cos(\theta_{M4}) + P_v = 322kip \cdot \cos(45deg) + 160kip = 388kip$$

Calculate moment M_Q acting at Q_{WP}

$$M_Q = P_v \cdot \left(\frac{L_v}{2} + e_{h.wp} - e_{Q.wp}\right) - V_v \cdot \frac{L_Q}{2} + F_{RCC.M3} \cdot e_{M3} - F_{RCC.M4} \cdot \sin(\theta_{M4}) \cdot e_{M4}$$

$$M_Q = 160kip \cdot \left(\frac{18.0in}{2} + 10.8in - 10.5in\right) - 160kip \cdot \frac{41.2in}{2} + 90kip \cdot 9.8in - 322kip \cdot \sin(45deg) \cdot 0.7in$$

$$M_Q = -1094kip \cdot in$$

Determine section modulus and calculate bending and normal stresses

$$S = \frac{L_Q^2 \cdot t}{6} = \frac{(41.2in)^2 \cdot 0.5in}{6} = 141in^3$$

$$\sigma_P = \frac{P_Q}{L_Q \cdot t} = \frac{158kip}{41.2in \cdot 0.5in} = 7.65ksi$$

$$\sigma_M = \frac{|M_Q|}{S} = \frac{|-1094kip \cdot in|}{141in^3} = 7.74ksi$$

Since $\sigma_P + \sigma_M < F_y$ and $\sigma_M > \sigma_P$, use σ in von Mises equation based on $0.6*\sigma_{max}$ (Refer to Appendix A)

$$\sigma_{0.6} = 0.6 \cdot (\sigma_P + \sigma_M) = 0.6 \cdot (7.65ksi + 7.76ksi) = 9.25ksi$$

$$\Omega = \sqrt{1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2} = \sqrt{1 - \left(\frac{9.25ksi}{36.4ksi}\right)^2} = 0.97$$

$$\tau_N = \Omega \cdot (0.58) \cdot F_y = 0.97 \cdot (0.58) \cdot 36.4ksi = 20.4ksi$$

Check shear on Section Q to see if it is less than 20.4 ksi

$$v_Q = \frac{V_Q}{L_Q \cdot t} = \frac{388kip}{41.2in \cdot 0.5in} = 18.8ksi \quad \leq \quad \tau_N = 20.4\,ksi$$

Therefore, remaining portion of gusset plate can sustain the demands of the Refined Corner Check

G1.2.3d RCC Buckling Check: Parallel Results

Check buckling due to axial forces on corner surfaces with Refined Corner Check demands (refer to Appendix B)

G1.2.3d1 Short Span Buckling Check:

For this gusset plate, the short span corresponds to the horizontal surface

$F_{cr} = 28.7\text{ksi}$ See Basic Corner Check

$\sigma = \sigma_h = 18.2\text{ksi}$

$\tau = \tau_h = 18.2\text{ksi}$

$$\sigma_{Princ} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{18.2\text{ksi}}{2} + \sqrt{\left(\frac{18.2\text{ksi}}{2}\right)^2 + (18.2\text{ksi})^2} = 29.5\text{ksi} \geq \quad F_{cr} = 28.7\text{ksi}$$

The principle stress due to the Refined Corner Check demands is greater than the critical buckling stress; therefore, must decrease calculated strength based on buckling capacity.

G1.2.3d2 Long Span Buckling Check:

Treat as flat rectangular plate with one non-loaded edge fixed and the remaining edges clamped

Not a concern as $a/b \leq 0.75$ See Basic Corner Check

Long Span Length (Figure 8)

$a = a_v = 8.3\text{in}$

Length of Long Side Surface (Figure 8)

$b = L_v = 18.0\text{in}$

Reduce capacity due to buckling by reducing input forces by the ratio of the overstress to determine available Refined Corner Check capacity with Parallel Resultants.

$$\text{Ratio} = \frac{F_{cr}}{\sigma_{Princ}} = \frac{28.7\text{ksi}}{29.5\text{ksi}} = 97.3\%$$

$C_{RCC} = C_{RCC} \cdot \text{Ratio} = 455\text{kip} \cdot 97.3\% = 443\text{kip}$

RCC Parallel Resultants Capacity (per plate)

Total member capacity
$2 \cdot 443\text{kip} = 886\text{ kip}$

$$ORF_{RCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{443\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -386\text{kip}\right|}{1.3 \cdot |-165\text{kip}|} = 1.79$$

$$IRF_{RCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{443\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -386\text{kip}\right|}{2.17 \cdot |-165\text{kip}|} = 1.07$$

G1.2.4 Refined Corner Check: Nonparallel Resultants

Removing the constraint that the corner surface resultants remain parallel can result in further optimization of the shear and normal forces on the surfaces and an increase in capacity. However, recognize that only a small capacity increase can be gained by further refinement of the analysis before Horizontal Shear controls and that the buckling strength of the short span (horizontal surface) is at capacity.

Allowing the surface resultants to be nonparallel creates multiple equations with multiple unknowns, requiring a complex iterative approach to achieve a solution. In selecting trial values for V and P, recognize that adjustments in shear have a 3x effect on shear stress when considering von Mises stress.

Because the horizontal surface controlled over the vertical surface when the resultants were parallel, maximize the load on the horizontal surface by adjusting the combination of shear and normal stresses. Thus, a decrease in V_h will increase the capacity and increase the shear on the stub (caused by increasing P_v). As a first iteration, select a V_h such that the final capacity is larger than the Horizontal Shear capacity.

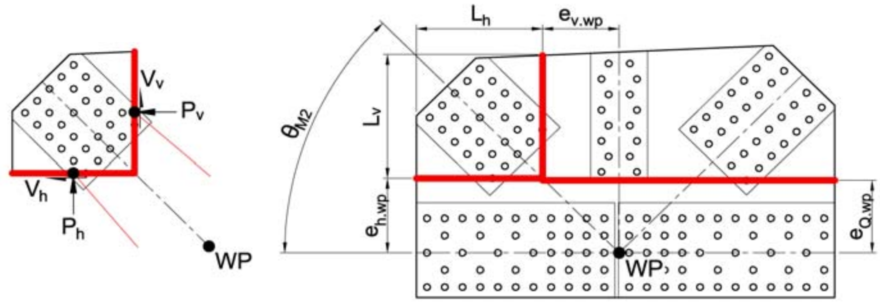


Figure 13: Refined Corner Check for Diagonal Member M2

G1.2.4a Determine Trial Forces and Overall Capacity with All Forces a Function of V_h:

G1.2.4a1 - Horizontal Surface:

Solve the von Mises stress relationship for the axial force on the horizontal surface so that P_h is a function of V_h

$$F_y^2 = \sigma^2 + 3\cdot\tau^2$$

$$F_y^2 = \left(\frac{P_h}{L_{h.}\cdot t}\right)^2 + 3\cdot\left(\frac{V_h}{L_{h.}\cdot t}\right)^2$$

$$P_h = \sqrt{F_y^2\cdot L_h^2\cdot t^2 - 3\cdot V_h^2}$$

G1.2.4a2 - Vertical Surface:

Solve for the forces acting on the vertical surface as a function of the forces acting on the horizontal surface

Constrain final resultant to be parallel to member to avoid bending in member

$$\operatorname{atan}\left(\frac{P_h + V_v}{V_h + P_v}\right) = \theta_{M2}$$

Constrain moments about work point to balance (i.e. $\Sigma M_{WP} = 0$)

$$\sum M = 0 = P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) - V_h \cdot e_{h.wp} - P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right) + V_v \cdot e_{v.wp}$$

Solve the two equations for P_v and V_v

$$P_v = \frac{P_h + V_v}{\tan\left(\theta_{M2}\right)} - V_h$$

$$V_v = \frac{P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right) + V_h \cdot e_{h.wp} - P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right)}{e_{v.wp}}$$

Substitute for P_v and V_v combine terms and simplify

$$P_v = \frac{P_h + \frac{P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right) + V_h \cdot e_{h.wp} - P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right)}{e_{v.wp}}}{\tan\left(\theta_{M2}\right)} - V_h$$

$$P_v = \frac{L_h \cdot P_h - 2 \cdot V_h \cdot e_{h.wp} + 2 \cdot V_h \cdot e_{v.wp} \cdot \tan\left(\theta_{M2}\right)}{L_v + 2 \cdot e_{h.wp} - 2 \cdot e_{v.wp} \cdot \tan\left(\theta_{M2}\right)}$$

G1.2.4a3 - Trial Force Substitution:

Choose a value for the shear on the horizontal surface (V_h) that gives a calculated capacity just above that of Horizontal Shear.

Recall: $C_{HS.M2} = 460\,kip$ Therefore, select $V_h = 155\,kip$

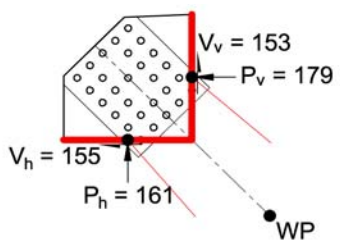
Solve for the following:

$$P_h = \sqrt{F_y^2 \cdot L_h^2 \cdot t^2 - 3 \cdot V_h^2} = \sqrt{(36.4ksi)^2 \cdot (17.8in)^2 \cdot (0.5in)^2 - 3 \cdot (155kip)^2} = 181kip$$

$$P_v = \frac{17.8in \cdot 181kip - 2 \cdot 155kip \cdot 10.8in + 2 \cdot 155kip \cdot 10.8in \cdot \tan(45deg)}{18in + 2 \cdot 10.8in - 2 \cdot 10.8in \cdot \tan(45deg)} = 179kip$$

$$V_v = \frac{179kip \cdot \left(\frac{18in}{2} + 10.8in\right) + 155kip \cdot 10.8in - 181\,kip\left(\frac{17.8in}{2} + 10.8in\right)}{10.8in} = 153kip$$

$$C_{RCC} = \sqrt{\left(V_h + P_v\right)^2 + \left(V_v + P_h\right)^2} = \sqrt{(155kip + 179kip)^2 + (153kip + 181kip)^2} = 472kip$$



RCC Nonparallel Resultants Capacity (per plate)

Total member capacity
$2 \cdot 472kip = 944\,kip$

Figure 14: Refined Corner Check Resultants with Resultants Not Parallel to Member

If the stress checks are adequate, this combination of forces will give a capacity just greater than that calculated by Horizontal Shear. Proceed knowing that the horizontal surface already is at maximum capacity and does not need to be checked.

G1.2.4b Vertical Surface Check: Nonparallel Resultants

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{179kip}{(18.0in) \cdot (0.5in)} = 19.9ksi \qquad \tau_v = \frac{V_v}{L_v \cdot t} = \frac{155kip}{(18.0in) \cdot (0.5in)} = 17.0ksi$$

$$\sigma_{vm.v} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{(19.9ksi)^2 + 3 \cdot (17.0ksi)^2} = 35.5ksi \quad \le \quad F_y = 36.4\,ksi$$

G1.2.4c Remaining Portion (Stub) Check: Nonparallel Resultants

Calculate equivalent concurrent forces for vertical and tension diagonal

$$F_{RCC.M3} = OpForce_{M3} \cdot \left| \frac{C_{RCC}}{OpForce_{M2}} \right| = 141\text{kip} \cdot \left| \frac{472\text{kip}}{-716\text{kip}} \right| = 93\text{kip}$$

$$F_{RCC.M4} = OpForce_{M4} \cdot \left| \frac{C_{RCC}}{OpForce_{M2}} \right| = 507\text{kip} \cdot \left| \frac{472\text{kip}}{-716\text{kip}} \right| = 334\text{kip}$$

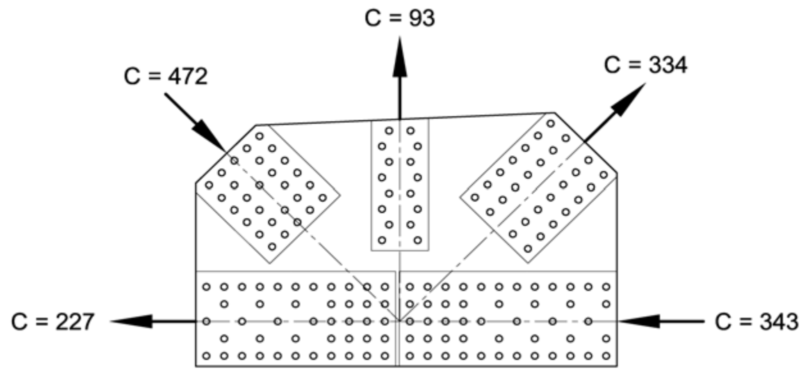


Figure 15: Concurrent Member Capacities (per plate) Based on Refined Corner Check (Subject to Stub Check and Buckling Check)

Check remaining portion of the gusset plate outside of the corner and chord. Select a Section Q that encompasses all forces applied by members M3 and M4.

$L_Q = 41.2\text{in}$

$e_{Q.wp} = 10.5\text{ in}$

$\theta_{M4} = 45 \cdot \text{deg}$

$e_{M3} = 9.8\text{ in}$

$e_{M4} = 0.7\text{ in}$

$L_v = 18\text{ in}$

$e_{h.wp} = 10.8\text{ in}$

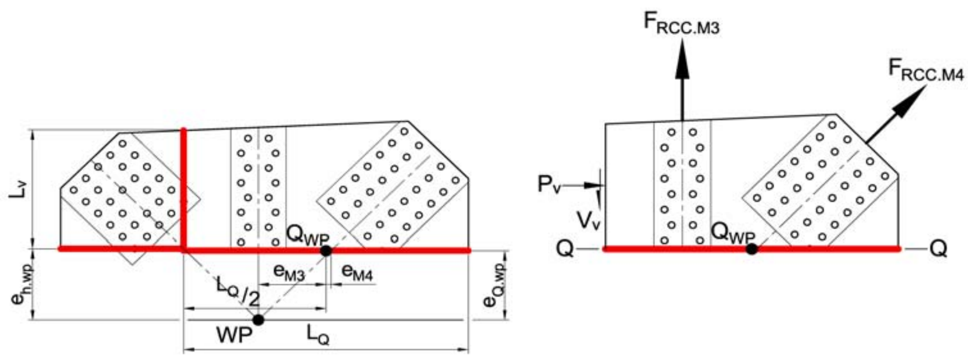


Figure 16: Remaining Gusset Plate Stub

G1.2.4c Remaining Portion (Stub) Check Cont.: Nonparallel Resultants

Calculate forces P_Q and V_Q along Section Q

$$P_Q = F_{RCC.M3} + F_{RCC.M4} \cdot \sin(\theta_{M4}) - V_v = 93\text{kip} + 334\text{kip} \cdot \sin(45\text{deg}) - 153\text{kip} = 177\text{kip}$$

$$V_Q = 0 + F_{RCC.M4} \cdot \cos(\theta_{M4}) + P_v = 0 + 334\text{kip} \cdot \cos(45\text{deg}) + 179\text{kip} = 415\text{kip}$$

Calculate moment M_Q about Section Q

$$M_Q = P_v \cdot \left(\frac{L_v}{2} + e_{h.wp} - e_{Q.wp}\right) - V_v \cdot \frac{L_Q}{2} + F_{RCC.M3} \cdot e_{M3} - F_{RCC.M4} \cdot \sin(\theta_{M4}) \cdot e_{M4}$$

$$M_Q = 179\text{kip} \cdot \left(\frac{18.0\text{in}}{2} + 10.8\text{in} - 10.5\text{in}\right) - 153\text{kip} \cdot \frac{41.2\text{in}}{2} + 93\text{kip} \cdot 9.8\text{in} - 334\text{kip} \cdot \sin(45\text{deg}) \cdot 0.7\text{in}$$

$$M_Q = -749\text{kip} \cdot \text{in}$$

Determine section modulus and calculate bending and normal stresses

$$S = \frac{L_Q^{\,2} \cdot t}{6} = \frac{(41.2\text{in})^2 \cdot 0.5\text{in}}{6} = 141\text{in}^3$$

$$\sigma_P = \frac{P_Q}{L_Q \cdot t} = \frac{177\text{kip}}{41.2\text{in} \cdot 0.5\text{in}} = 8.57\text{ksi}$$

$$\sigma_M = \frac{|M_Q|}{S} = \frac{|-749\text{kip} \cdot \text{in}|}{141\text{in}^3} = 5.30\text{ksi}$$

Since $\sigma_P + \sigma_M < F_y$ and $\sigma_M < \sigma_P$, use σ in von Mises equation based on σ at 0.6*L (Refer to Appendix A)

$$\sigma_{0.6} = (\sigma_P - \sigma_M) + 0.6 \cdot \left[(\sigma_P + \sigma_M) - (\sigma_P - \sigma_M)\right]$$

$$\sigma_{0.6} = (8.57\text{ksi} - 5.30\text{ksi}) + 0.6 \cdot [(8.57\text{ksi} + 5.30\text{ksi}) - (8.57\text{ksi} - 5.30\text{ksi})] = 9.63\text{ksi}$$

$$\Omega = \sqrt{1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2} = \sqrt{1 - \left(\frac{9.63\text{ksi}}{36.4\text{ksi}}\right)^2} = 0.96$$

$$\tau_N = \Omega \cdot (0.58) \cdot F_y = 0.96 \cdot (0.58) \cdot 36.4\text{ksi} = 20.4\text{ksi}$$

Check shear on Section Q to see if it is less than 20.4 ksi

$$v_Q = \frac{V_Q}{L_Q \cdot t} = \frac{415\text{kip}}{41.2\text{in} \cdot \frac{1}{2}\text{in}} = 20.2\text{ksi} \qquad \le \qquad \tau_N = 20.4\text{ ksi}$$

Therefore, remaining portion of gusset plate can sustain the demands of the Refined Corner Check.

G1.2.4d Buckling Check: Nonparallel Resultants

Check buckling due to axial forces on surfaces (refer to Appendix B)

G1.2.3d1 Short Span Buckling Check:

For this gusset plate, the short span corresponds to the horizontal surface

$F_{cr} = 28.7ksi$ See Basic Corner Check

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{181kip}{(17.8in)\cdot(0.5in)} = 20.4ksi \qquad \tau_h = \frac{V_h}{L_h \cdot t} = \frac{155kip}{(17.8in)\cdot(0.5in)} = 17.4ksi$$

$$\sigma_{Princ} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{20.4ksi}{2} + \sqrt{\left(\frac{20.4ksi}{2}\right)^2 + (17.4ksi)^2} = 30.4ksi \quad \geq \quad F_{cr} = 28.7ksi$$

The principle stress is greater than the critical buckling stress; therefore, decrease calculated strength based on buckling capacity.

G1.2.3d1 Long Span Buckling Check:

Treat as flat rectangular plate with one non-loaded edge fixed and the remaining edges clamped

Not a concern as $a/b \leq 0.75$ See Basic Corner Check

Long Span Length (Figure 8)

$a = a_v = 8.3in$

Length of Long Side Surface (Figure 8)

$b = L_v = 18.0in$

Reduce capacity due to buckling by reducing input forces by the ratio of the overstress to determine available Refined Corner Check capacity with nonparallel resultants.

$$Ratio = \frac{F_{cr}}{\sigma_{Princ}} = \frac{28.7ksi}{30.4ksi} = 94.4\%$$

$$C_{RCC} = C_{RCC} \cdot Ratio = 472kip \cdot 94.4\% = 446kip$$

RCC Nonparallel Resultants Capacity (per plate)

This value is only slightly better than the RCC-Parallel case

Total member capacity
$2 \cdot 446kip = 892\ kip$

$$ORF_{RCC} = \frac{C_{RCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{446kip - 1.3 \cdot \left|\frac{1}{2} \cdot -386kip\right|}{1.3 \cdot |-165kip|} = 1.82$$

$$IRF_{RCC} = \frac{C_{RCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{446kip - 1.3 \cdot \left|\frac{1}{2} \cdot -386kip\right|}{2.17 \cdot |-165kip|} = 1.09$$

This solution represents only a minimal increase over when the resultants are parallel. Additional iterations could be carried out; however, any potential increase in capacity is limited to 3% before the Horizontal Shear capacity controls

G1.2.5 Evaluation Summary:

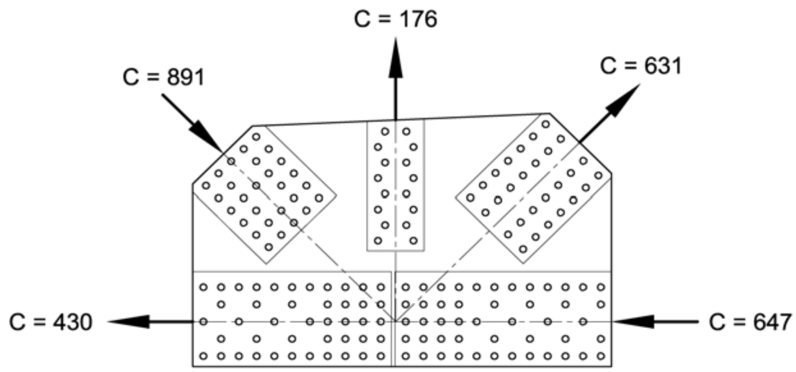


Figure 17: Concurrent Member Capacities Based on Refined Analysis (for Gusset Plate Pair)

Limit State	Gusset Plate Pair	
	Operating Rating	Inventory Rating
Fasteners	4.42	2.65
Vertical Shear	4.08	2.44
~~Horizontal Shear~~[1]	~~1.89~~	~~1.13~~
~~Partial Shear Yield~~[2]	~~0.61~~	~~0.37~~
~~Whitmore Compression~~[2]	~~0.95~~	~~0.57~~
Tension	4.32	2.59
Block Shear	4.18	2.51
Chord Splice	16.10	9.64
Horizontal Shear (Ω Calc.)	1.95	1.16
~~Basic Corner Check~~[3]	~~1.28~~	~~0.77~~
Refined Corner Check	1.82	1.09

Controls

[1] Superceded by Horizontal Shear with Ω calculated.

[2] Superceded by Basic Corner Check (see [3]).

[3] Superceded by final iteration of Refined Corner Check.

By refining the analysis calculations using the approach presented above, a nearly 200% increase in the Operating Rating can be achieved.

Example 2 - Noncompact Gusset Plate with Long Vertical Buckling Length

Load Factor Rating (LFR) Method

Example 2 is a four member gusset plate (no vertical) with a relatively long buckling length between diagonals. It is not a compact gusset plate and no members are chamfered. Calculations apply to one of two gusset plates.

G2.1 Gusset Plate Material, Geometric, and Loading Properties:

Material Properties

$F_y = 36.4\text{ksi}$

$F_u = 62.6\text{ksi}$

$E = 29000\text{ksi}$

$\nu = 0.3$

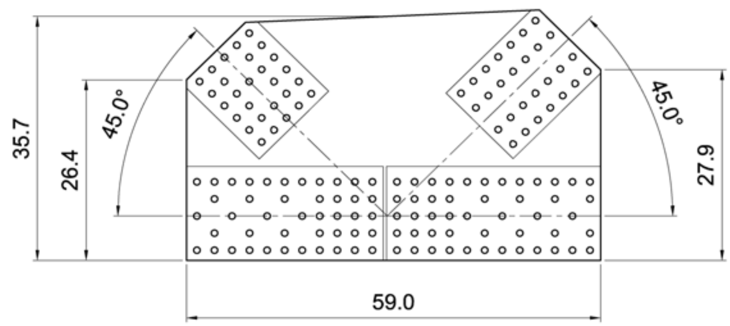


Plate Thickness

$t = \frac{7}{16}\text{in} = 0.4375\text{ in}$

Member Angles

Figure 1: Basic Geometry of Gusset Plate

$\theta_{M2} = 45\text{deg}$

$\theta_{M4} = 45\text{deg}$

Unfactored Member Forces Per Gusset Plate Pair

$LL_{M1} = 190\text{ kip}$

$DL_{M1} = 190\text{ kip}$

$LL_{M2} = -275\text{ kip}$

$DL_{M2} = -275\text{ kip}$

$LL_{M4} = 275\text{ kip}$

$DL_{M4} = 275\text{ kip}$

$LL_{M5} = -200\text{ kip}$

$DL_{M5} = -200\text{ kip}$

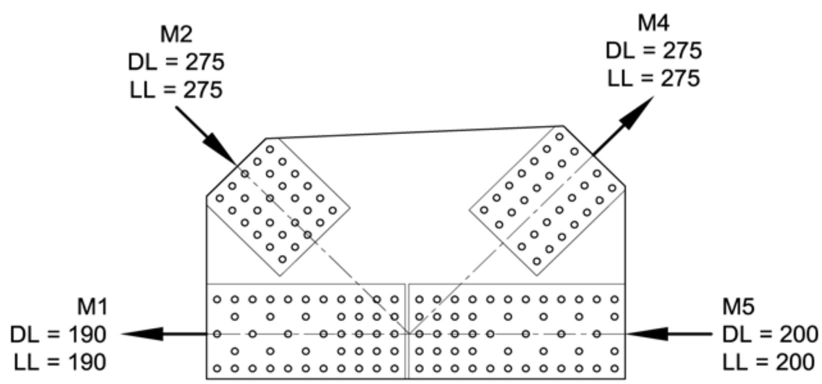


Figure 2: Concurrent Member Forces Transferred to Two Gusset Plates

Member forces based on NCHRP Project 12-84 loads with an assumed Dead Load to Live Load ratio of 50/50.

G2.1 Gusset Plate Material, Geometric, and Loading Properties Cont.:

Factored Forces Acting on Gusset Plate Pair

$InvForce_{M1} = \gamma_{InvLL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 2.17 \cdot 190kip + 1.3 \cdot 190kip$ $\quad InvForce_{M1} = 658\ kip$

$OpForce_{M1} = \gamma_{LL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 1.3 \cdot 190kip + 1.3 \cdot 190kip$ $\quad OpForce_{M1} = 493\ kip$

$InvForce_{M2} = -956\ kip$

$OpForce_{M2} = -716\ kip$

$InvForce_{M4} = 956\ kip$

$OpForce_{M4} = 716\ kip$

$InvForce_{M5} = -694\ kip$

$OpForce_{M5} = -520\ kip$

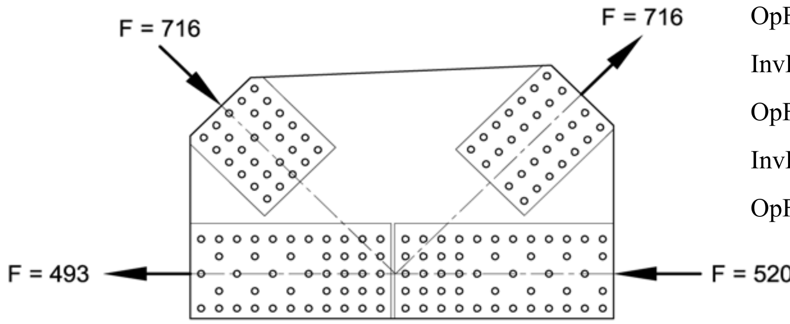


Figure 3: Concurrent Member Operating Forces Transferred to Two Gusset Plates

G2.2 Evaluation Approach:

In accordance with the *2014 Interim Revisions to the Manual for Bridge Evaluation, Second Edition,* the following gusset plate limit state checks were done:
(a) Fastener strength (L6B.2.6.1)
(b) Vertical shear resistance (L6B.2.6.3)
(c) Horizontal shear resistance (L6B.2.6.3)
(d) Partial shear yield resistance (L6B.2.6.3)
(e) Compressive (Whitmore) resistance (L6B.2.6.4)
(f) Tension strength (L6B.2.6.5)
(g) Bock shear resistance (L6B.2.6.5)
(h) Chord splice capacity (L6B.2.6.6)

Load Factor Rating Summary for Example 2

Limit State	Gusset Plate Pair		
	Operating Rating	Inventory Rating	
Fasteners	3.05	1.83	7/8 in. diam A325 threads excluded fasteners
Vertical Shear	2.45	1.47	Ω = 0.88 with splice plates included
Horizontal Shear	0.89	0.54	
Partial Shear Yield	0.68	0.41	**Controls**
Whitmore Compression	0.69	0.41	
Tension	1.48	0.89	
Block Shear	1.43	0.85	
Chord Splice	107.0	64.3	

When the Partial Shear Plane Yield and/or Whitmore Compression capacity checks control and indicate a less than acceptable rating, more rigorous evaluation should be performed.

The following more rigorous rating checks are performed in Example 2:
(1) Horizontal shear capacity - Ω calculated: Supercedes Horzizontal Shear with Ω = 0.88.
(2) Basic Corner Check capacity (BCC): Replaces Partial Shear Plane Yield and Whitmore Compression capacity che
(3) Refined Corner Check capacity (RCC): Supercedes BCC unless BCC indiates acceptable rating.

G2.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω):

Global shear check along horizontal plane parallel with bottom chord. Shear force calculated using horizontal component of diagonal member forces. Gross section selected at bottom fastener of diagonal members to achieve maximum eccentricity. Net section calculated through bottom chord fastener holes. Ω calculated using Drucker formula.

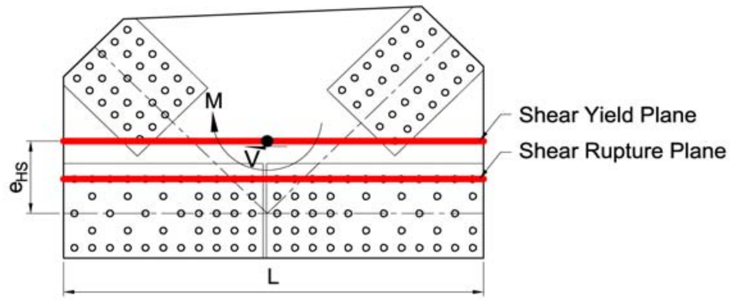


Figure 4: Horizontal Shear Between Web and Chord Members

$L = 59.0\,\text{in}$

$e_{HS} = 10.52\,\text{in}$

$M = V \cdot e_{HS}$

$A_g = t \cdot L_{HS} = 0.4375\text{in} \cdot 59.0\text{in} = 25.8\text{in}^2$

$d_h = 1\,\text{in}$

$n_{hole} = 23$

$A_n = t \cdot \left(L - n_{hole} \cdot d_h\right) = 0.4375\text{in} \cdot [59.0\text{in} - (23) \cdot 1.0\text{in}] = 15.8\text{in}^2$

Calculate Ω using Drucker formula instead of using AASHTO-specified Ω=0.88

$$V = V_p \cdot \left[1 - \left(\frac{M}{M_p}\right)\right]^{0.25} \qquad V = \Omega \cdot V_p$$

Drucker Formula [1]

$$V_P = (0.58) \cdot F_y \cdot A_g = (0.58) \cdot 36.4\text{ksi} \cdot 25.8\text{in}^2 = 545\text{kip}$$

$$M_P = \frac{L^2 \cdot t}{4} \cdot F_y = \frac{(59.0\text{in})^2 \cdot 0.4375\text{in}}{4} \cdot 36.4\text{ksi} = 13900\text{in} \cdot \text{kip}$$

Substitute $V = \Omega * V_p$ into Drucker formula and rearrange to solve for Ω using plastic shear and moment

$$\Omega \cdot V_p = V_p \left(1 - \frac{\Omega \cdot V_P \cdot e_{HS}}{M_P}\right)^{0.25} = 0.89$$

$$\Omega = \left(1 - \frac{\Omega \cdot V_P \cdot e_{HS}}{M_P}\right)^{0.25} = \left(1 - \frac{\Omega \cdot 545\text{kip} \cdot 10.52\text{in}}{13900\text{in} \cdot \text{kip}}\right)^{0.25} = 0.89$$

Requires iterative process since V is proportional to Ω. Can substitute AASHTO specified value of Ω = 0.88 on right side of equation as a first estimate of Ω. Result shown is the calculated value of Ω after performing necessary iterations.

G2.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω) Cont.:

$\phi_{vy} = 1.0$

$\phi_{vu} = 0.85$

$C_Y = \varphi_{yy} \cdot (0.58) \cdot F_y \cdot A_g \cdot \Omega = 1.00(0.58) \cdot 36.4\text{ksi} \cdot 25.8\text{in}^2 \cdot (0.89) = 486\text{kip}$

$C_U = \varphi_{yu} \cdot (0.58) \cdot F_u \cdot A_n = 0.85(0.58) \cdot 62.6\text{ksi} \cdot 15.8\text{in}^2 = 486\text{kip}$

$C_{HS} = \min\left(C_Y, C_U\right) = \min(486\text{kip}, 486\text{kip}) = 486\text{kip}$

Horizontal Shear Capacity (per plate)

Total member capacity

$2 \cdot 485\text{kip} = 971\text{kip}$

Determine capacity of member M2 based on Horizontal Shear

$$C_{HS.M2} = C_{HS} \cdot \left| \frac{OpForce_{M2}}{OpForce_{M1} - OpForce_{M5}} \right| = 486\text{kip} \cdot \left| \frac{-716\text{kip}}{493\text{kip} - (-520\text{kip})} \right| = 343\text{kip}$$

$$ORF_{HS} = \frac{C_{HS.M2} - \gamma_{DL} \cdot \left| \frac{1}{2} \cdot DL_{M2} \right|}{\gamma_{LL} \cdot \left| LL_{M2} \right|} = \frac{343\text{kip} - 1.3 \cdot \left| \frac{1}{2} \cdot -275\text{kip} \right|}{1.3 \cdot \left| \frac{1}{2} \cdot -275\text{kip} \right|} = 0.92$$

$$IRF_{HS} = \frac{C_{HS.M2} - \gamma_{DL} \cdot \left| \frac{1}{2} \cdot DL_{M2} \right|}{\gamma_{InvLL} \cdot \left| LL_{M2} \right|} = \frac{343\text{kip} - 1.3 \cdot \left| \frac{1}{2} \cdot -275\text{kip} \right|}{2.17 \cdot \left| \frac{1}{2} \cdot -275\text{kip} \right|} = 0.55$$

[1] Drucker, D., *The Effect of Shear on the Plastic Bending of Beams*, American Society of Mechanical Engineers, NAMD Conference, Urbana, IL, June 1956

G2.2.2 Basic Corner Check:

The Basic Corner Check is a first-principles analytical approach utilizing fundamental steel design theory to conservatively calculate gusset plate limit state capacities at critical cross sections. This check is used to evaluate equilibrium and stability of a gusset plate "corner" bounded by horizontal and vertical planes that create the smallest section encompassing all fasteners of the diagonal member. The diagonal member force is assumed to be resisted by a combination of shear and normal forces acting on the vertical and horizontal surfaces bounding the "corner". Von Mises stress calculated on the surfaces is limited to the yield strength of the gusset plate. For simplicity and to avoid bending in the members, the resultant of each surface must pass through the work point. The "corner" can be adjusted in terms of location and plate thickness to accomodate deterioration.

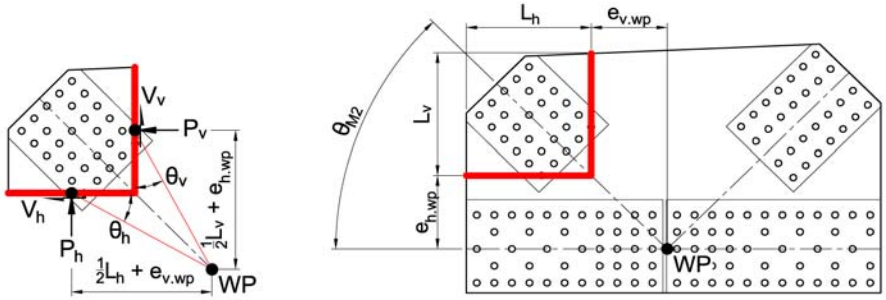


Figure 5: Basic Corner Check for Diagonal Member M2

Calculate resultant angles from the work point

$L_h = 17.8\,\text{in}$ $\qquad e_{h.wp} = 10.8\,\text{in}$

$L_v = 18\,\text{in}$ $\qquad e_{v.wp} = 10.8\,\text{in}$

$$\theta_h = \text{atan}\left(\frac{e_{h.wp}}{\frac{L_h}{2} + e_{v.wp}}\right) = \text{atan}\left(\frac{10.8\text{in}}{\frac{17.8\text{in}}{2} + 10.8\text{in}}\right) = 28.7\text{deg}$$

$$\theta_v = \text{atan}\left(\frac{e_{v.wp}}{\frac{L_v}{2} + e_{h.wp}}\right) = \text{atan}\left(\frac{10.8\text{in}}{\frac{18.0\text{in}}{2} + 10.8\text{in}}\right) = 28.6\text{deg}$$

G2.2.2a Horizontal Surface Check:

Since $L_h < L_v$ set von Mises stress on horizontal sruface equal to plate yield strength. After stresses on both surfaces are determined, verify assumption that horizontal surface is critical (i.e. reaches von Mises yield before vertical surface).

$$P_h = V_h \cdot \tan(\theta_h)$$

$$\sigma_h = \frac{P_h}{A_h} = \frac{P_h}{L_h \cdot t}$$

$$\tau_h = \frac{V_h}{A_h} = \frac{V_h}{L_h \cdot t}$$

$$\sigma_{vm} = \sqrt{\sigma_h^{\ 2} + 3\tau_h^{\ 2}}$$

Substitute P_h as a function of V_h and set the von Mises stress to yield

$$F_y = 36.4\text{ksi} = \sigma_{vm} = \sqrt{\sigma_h^{\ 2} + 3\tau_h^{\ 2}} = \sqrt{\left(\frac{P_h}{L_h \cdot t}\right)^2 + 3 \cdot \left(\frac{V_h}{L_h \cdot t}\right)^2} = \sqrt{\left(\frac{V_h \cdot \tan(\theta_h)}{L_h \cdot t}\right)^2 + 3 \cdot \left(\frac{V_h}{L_h \cdot t}\right)^2}$$

Rearrange terms and solve for V_h

$$V_h = \frac{L_h \cdot F_y \cdot t}{\sqrt{\tan(\theta_h)^2 + 3}} = \frac{17.8\text{in} \cdot 36.4\text{ksi} \cdot 0.4375\text{in}}{\sqrt{\tan(28.7\text{deg})^2 + 3}} = 156\text{kip}$$

Solve for P_h

$$P_h = V_h \cdot \tan(\theta_h) = 156\text{kip} \cdot \tan(28.7\text{deg}) = 86\text{kip}$$

Calculate shear and normal stresses on horizontal surface

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{86\text{kip}}{(17.8\text{in}) \cdot (0.4375\text{in})} = 11.0\text{ksi} \qquad \tau_h = \frac{V_h}{L_h \cdot t} = \frac{156\text{kip}}{(17.8\text{in}) \cdot (0.4375\text{in})} = 20.0\text{ksi}$$

G2.2.2b Vertical Surface Check:

Determine forces and stresses on vertical surface based on horizontal surface forces and stanted constraints (i.e. force resultants to pass thru workpoint).

$$P_v = V_v \cdot \tan(\theta_v)$$

$$\theta_v = 28.6 \cdot deg$$

$$\theta_{M2} = atan\left(\frac{V_v + P_h}{P_v + V_h}\right)$$

Substitue P_v as a function of V_v

$$\theta_{M2} = atan\left(\frac{V_v + P_h}{P_v + V_h}\right) = atan\left(\frac{V_v + P_h}{V_v \cdot \tan(\theta_v) + V_h}\right)$$

Rearrange terms and solve for V_v. Substitute values obtained from previously solving P_h and V_h.

$$V_v = \frac{P_h - V_h \cdot \tan(\theta_{M2})}{\tan(\theta_{M2}) \cdot \tan(\theta_v) - 1} = \frac{86kip - 156kip \cdot \tan(45deg)}{\tan(45deg) \cdot \tan(28.6deg) - 1} = 155kip$$

Solve for P_v

$$P_v = V_v \cdot \tan(\theta_v) = 155kip \cdot \tan(28.6deg) = 84kip$$

Calculate shear and normal stresses on vertical surface

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{84kip}{(18.0in) \cdot (0.4375in)} = 10.7ksi \qquad \tau_v = \frac{V_v}{L_v \cdot t} = \frac{155kip}{(18.0in) \cdot (0.4375in)} = 19.7ksi$$

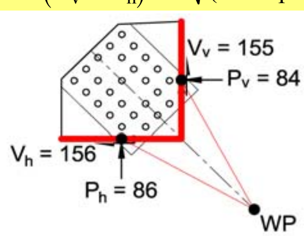
Calculate von Mises stress

$$\sigma_{vm.v} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{(10.7ksi)^2 + 3 \cdot (19.7ksi)^2} = 35.7ksi \qquad \le \qquad F_y = 36.4ksi$$

Since von Mises stress on vertical surface is less than yield strength of the gusset plate, the horizontal surface controls. If this had not been the case, the von Mises stress calcualted on the vertical surface would have been greater than the yield stress. The previous process would have been modified by first setting the von Mises stress on the vertical surface to the yield stress, and then determining the necessary resultants on the horizontal surface to balance the moment about the work point.

Substitute corresponding solved forces to determine member resultant force.

$$C_{BCCvM} = \sqrt{(V_h + P_v)^2 + (V_v + P_h)^2} = \sqrt{(156kip + 84kip)^2 + (155kip + 86kip)^2} = 340kip$$



BCC von Mises Capacity (per plate)

Total member capacity

$$2 \cdot 389kip = 778\,kip$$

Figure 6: Basic Corner Check Resultants for Diagonal Member M2

G2.2.2c BCC Buckling Check:

Check plate buckling due to axial forces on Basic Corner Check surfaces (refer to Appendix B). If buckling controls, then von Mises stresses must be adjusted.

Evaluate buckling capacity of plate regions defined by the short and long span between the Basic Corner Check surfaces and adjacent member connections.

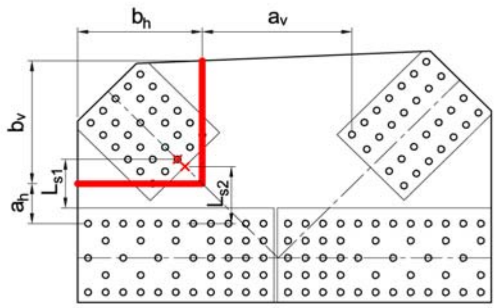


Figure 7: Corner Check Buckling Lengths

G2.2.2c1 Short Span Buckling Check:

For this gusset plate, the short span corresponds to the horizontal surface ($a_h < a_v$). a_h and a_v are defined as the distances from the respective Corner Check surface to the parallel line passing through the nearest fastener in an adjacent member.

$$L_s = \frac{L_{s1} + L_{s2}}{2} = \frac{7.1in + 8.3in}{2} = 7.7in$$

$$r = \frac{t}{\sqrt{12}} = \frac{0.438in}{\sqrt{12}} = 0.13in$$

Short span assumed fixed against rotation but free to translate at one end and retrained against translation and rotation at the other. Therefore K=1.0 used.

$$F_e = \frac{\pi^2 \cdot E}{\left(\frac{K \cdot L_s}{r}\right)^2} = \frac{\pi^2 \cdot 29000ksi}{\left[\frac{1.0 \cdot (7.7in)}{0.13in}\right]^2} = 77.0ksi$$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2 \cdot \sqrt{2}}\right) = 36.4ksi \cdot \left(1 - \frac{\sqrt{\frac{36.4ksi}{77ksi}}}{2 \cdot \sqrt{2}}\right) = 27.6ksi$$

$$\sigma = \sigma_h = 11.0ksi$$

$$\tau = \tau_h = 20.0ksi$$

$$\sigma_{Princ} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{11.0ksi}{2} + \sqrt{\left(\frac{11.0ksi}{2}\right)^2 + (20.0ksi)^2} = 26.3ksi \quad \leq \quad F_{cr} = 27.6ksi$$

Principle stress is less than the critical buckling stress; therefore, buckling of short span does not control

G2.2.2c2 Long Span Buckling Check:

Treat as flat rectangular plate with one non-loaded edge fixed and the remaining edges clamped (dashed curve D in Figure 8)

Long Span Length (Figure 8)

$$a = a_v = 21.3\text{in}$$

Length of Long Side Surface (Figure 8)

$$b = L_v = 18\text{in}$$

$$\frac{a}{b} = 1.18$$

Because a/b is greater than 0.75 (where k curve is nearly asymptotic), buckling of long span plate may be a concern. Therefore, calculate k as follows (using an approximate best fit function of dashed curve D in Figure 8):

$$k = 4.64 \cdot \left(\frac{a}{b}\right)^{-1.106} = 3.85$$

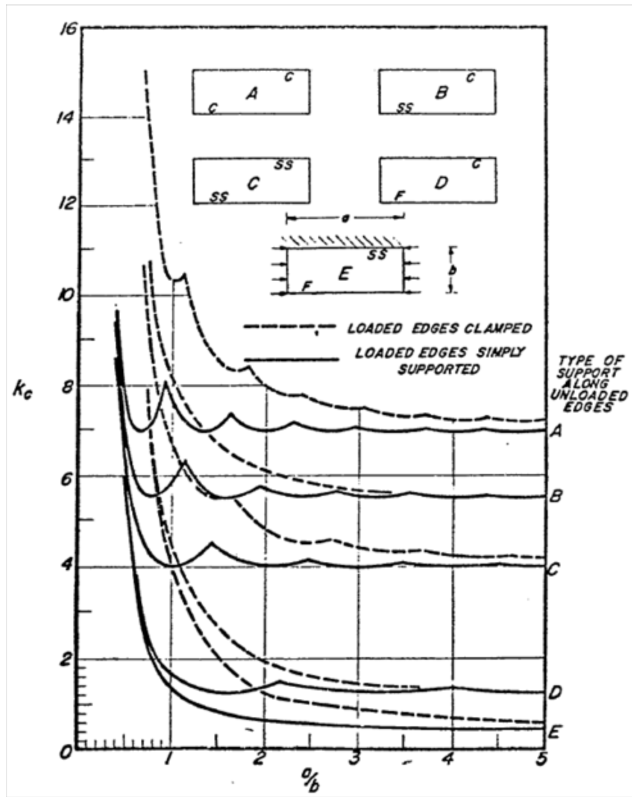


Figure 8: Elastic Buckling Coefficients [2]

$$F_e = \frac{k \cdot \pi^2 \cdot E}{12\left(1 - \nu^2\right) \cdot \left(\frac{b}{t}\right)^2} = \frac{3.85 \cdot \pi^2 \cdot 29000\text{ksi}}{12 \cdot \left(1 - 0.3^2\right) \cdot \left(\frac{18.0\text{in}}{0.4375\text{in}}\right)^2} = 59.5\text{ksi}$$

Since $\quad F_e = 59.5\text{ksi} > \frac{F_y}{2} = 18.2\text{ksi}$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2 \cdot \sqrt{2}}\right) = 36.4\text{ksi} \cdot \left(1 - \frac{\sqrt{\frac{36.4\text{ksi}}{59.5\text{ksi}}}}{2 \cdot \sqrt{2}}\right) = 26.3\text{ksi}$$

$$\sigma = \sigma_v = 10.7\text{ksi} \qquad \tau = \tau_v = 19.7\text{ksi}$$

$$\sigma_{Princ} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{10.7\text{ksi}}{2} + \sqrt{\left(\frac{10.7\text{ksi}}{2}\right)^2 + (19.7\text{ksi})^2} = 25.7\text{ksi} \quad \leq \quad F_{cr} = 26.3\text{ksi}$$

The principle stress is less than the critical buckling stress; therefore, buckling is not a concern.

G2.2.2 Basic Corner Check Cont.:

Since buckling of the short and long spans are not a concern for the Basic Corner Check, no reduction in calculated capacity is required, and capacity calculated using von Mises stress applies.

$C_{BCC} = 340\,\text{kip}$

BCC Resultant Capacity (per plate)

Total member capacity

$2 \cdot 340\text{kip} = 680\,\text{kip}$

$$ORF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{340\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -275\text{kip}\right|}{1.3 \cdot \left|-275\text{kip}\right|} = 0.90$$

$$IRF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{340\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -275\text{kip}\right|}{2.17 \cdot \left|-275\text{kip}\right|} = 0.54$$

If an increased rating factor is required, perform a Refined Corner Check.

[2] George Gerard and Herbert Becker. *Handbook of Structural Stability*, Part I - Buckling of Flat Plates, Tech. Note 3871, National Advisory Committee for Aeronautics, Washington, D.C., July 1957.

G2.2.3 Refined Corner Check:

The Refined Corner Check removes the constraint that surface resultants pass through the work point as assumed in the Basic Corner Check. In removing this constraint, it is important to check the portion of gusset plate outside of the corner (Stub) and check again for plate buckling based on these resultants.

An efficient initial starting point in this iterative check is to force the resultants acting on each surface to be parallel to the member and then adjust shear and normal forces as necessary.

G2.2.3a Horizontal Surface Check: Parallel Resultants

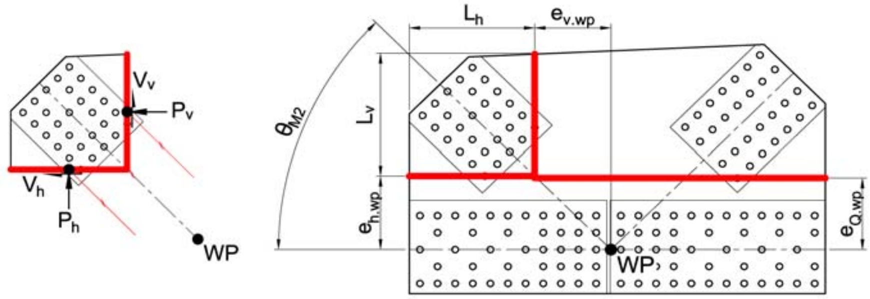


Figure 9: Refined Corner Check for Diagonal Member M2

As with the Basic Corner Check, check to see if the horizontal surface is the controlling surface by setting von Mises stress on horizontal sruface equal to plate yield strength. After stresses on both surfaces are determined; verify assumption that horizontal surface is critical (i.e. reaches von Mises yield before vertical surface).

$$V_h = \frac{P_h}{\tan(\theta_{M2})}$$

Constrain von Mises stress on surface equal to the plate yield stress

$$\sigma_{vm} = \sqrt{\sigma_h^2 + 3\tau_h^2} = F_y$$

$$L_h = 17.8\,in$$

$$\theta_{M2} = 45 \cdot deg$$

Substitute V_h as a function of P_h and set the von Mises stress to yield

$$F_y = 36.4ksi = \sigma_{vm} = \sqrt{\sigma_h^2 + 3\tau_h^2} = \sqrt{\left(\frac{P_h}{L_h \cdot t}\right)^2 + 3 \cdot \left(\frac{V_h}{L_h \cdot t}\right)^2} = \sqrt{\left(\frac{P_h}{L_h \cdot t}\right)^2 + 3 \cdot \left(\frac{\frac{P_h}{\tan(\theta_{M2})}}{L_h \cdot t}\right)^2}$$

Rearrange terms and solve for P_h

$$P_h = \frac{F_y \cdot L_h \cdot t \cdot \tan(\theta_{M2})}{\sqrt{\tan(\theta_{M2})^2 + 3}} = \frac{36.4ksi \cdot 17.8in \cdot 0.4375in \cdot \tan(45deg)}{\sqrt{\tan(45deg)^2 + 3}} = 142kip$$

Solve for V_h

$$V_h = \frac{P_h}{\tan(\theta_{M2})} = \frac{142kip}{\tan(45deg)} = 142kip$$

G2.2.3a Horizontal Surface Check Cont.: Parallel Resultants

Calculate resultants stresses on horizontal surface

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{142\text{kip}}{(17.8\text{in})\cdot(0.4375\text{in})} = 18.2\text{ksi} \qquad \tau_h = \frac{V_h}{L_h \cdot t} = \frac{142\text{kip}}{(17.8\text{in})\cdot(0.4375\text{in})} = 18.2\text{ksi}$$

G2.2.3b Vertical Surface Check: Parallel Resultants

Constrain moments about work point to balance (i.e. $\Sigma M_{WP} = 0$)

$$V_v = P_v \cdot \tan(\theta_{M2})$$

$$L_v = 18\,\text{in}$$

$$e_{v.wp} = 10.8\,\text{in}$$

$$e_{h.wp} = 10.8\,\text{in}$$

$$\sum M = 0 = \left[P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) - V_h \cdot e_{h.wp}\right] - \left[P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right) - V_v \cdot e_{v.wp}\right]$$

Substitute V_v as a function of P_v, rearrange terms, and solve for P_v

$$0 = \left[P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) - V_h \cdot e_{h.wp}\right] - \left[P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right) - P_v \cdot \tan(\theta_{M2}) \cdot e_{v.wp}\right]$$

$$P_v = \frac{\left[P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) - V_h \cdot e_{h.wp}\right]}{\left[\left(\frac{L_v}{2} + e_{h.wp}\right) - \tan(\theta_{M2}) \cdot e_{v.wp}\right]} = \frac{142\text{kip} \cdot \left(\frac{17.8\text{in}}{2} + 10.8\text{in}\right) - 142\text{kip} \cdot (10.8\text{in})}{\left(\frac{18\text{in}}{2} + 10.8\text{in}\right) - \tan(45\text{deg}) \cdot 10.8\text{in}} = 140\text{kip}$$

Solve for V_v

$$V_v = P_v \cdot \tan(\theta_{M2}) = 140\text{kip} \cdot \tan(45\text{deg}) = 140\text{kip}$$

Calculate resultants stresses on vertical surface

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{140\text{kip}}{(18.0\text{in})\cdot(0.4375\text{in})} = 17.8\text{ksi} \qquad \tau_v = \frac{V_v}{L_v \cdot t} = \frac{140\text{kip}}{(18.0\text{in})\cdot(0.4375\text{in})} = 17.8\text{ksi}$$

Calculate von Mises stress on vertical surface

$$\sigma_{vm.v} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{(17.8\text{ksi})^2 + 3\cdot(17.8\text{ksi})^2} = 35.5\text{ksi} \qquad \le \qquad F_y = 36.4\text{ksi}$$

G2.2.3 Refined Corner Check Cont.: Parallel Resultants

Since the von Mises stress on the vertical surface is less than the yield strength of the gusset plate, the horizontal surface controls, as assumed. If this had not been the case, the von Mises stress calcualted on the vertical surface would have been greater than the yield stress. The previous process would have been modified by first setting the von Mises stress on the vertical surface to the yield stress and then determining the necessary resultants on the horizontal surface to balance the moment about the work point.

$$C_{RCC} = \sqrt{\left(V_h + P_v\right)^2 + \left(V_v + P_h\right)^2} = \sqrt{(142\text{kip} + 140\text{kip})^2 + (140\text{kip} + 142\text{kip})^2} = 398\text{kip}$$

RCC Parallel Resultants Capacity (per plate)

Total member capacity

$2 \cdot 398\text{kip} = 796\text{ kip}$

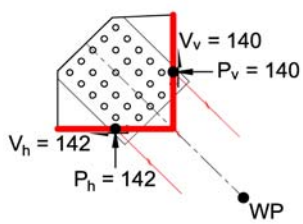


Figure 10: Refined Corner Check Resultants with Parallel Resultants to Member

G2.2.3c Remaining Portion (Stub) Check: Parallel Resultants

Determine equivalent concurrent force for tension diagonal per plate

$$F_{RCC.M4} = OpForce_{M4} \cdot \left| \frac{C_{RCC}}{OpForce_{M2}} \right| = 716kip \cdot \left| \frac{398kip}{-716kip} \right| = 398kip$$

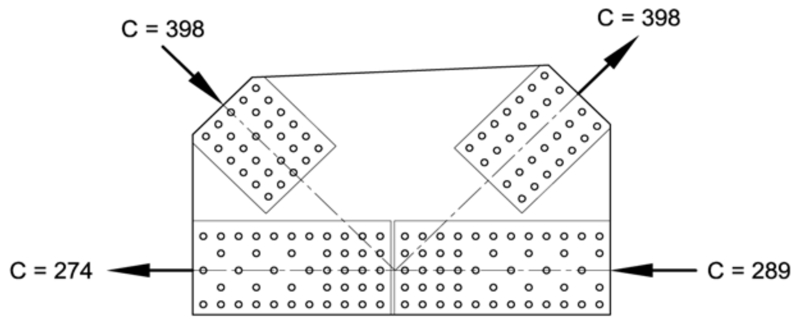


Figure 11: Concurrent Member Capacities (per plate) Based on Refined Corner Check (Subject to Stub Check and Buckling Check)

Check remaining portion of the gusset plate outside of the corner and chord. Select a Section Q that encompasses all forces applied by member M4.

$L_Q = 41.2in$

$e_{Q.wp} = 10.5\ in$

$\theta_{M4} = 45 \cdot deg$

$e_{M4} = 0.7\ in$

$L_v = 18\ in$

$e_{h.wp} = 10.8\ in$

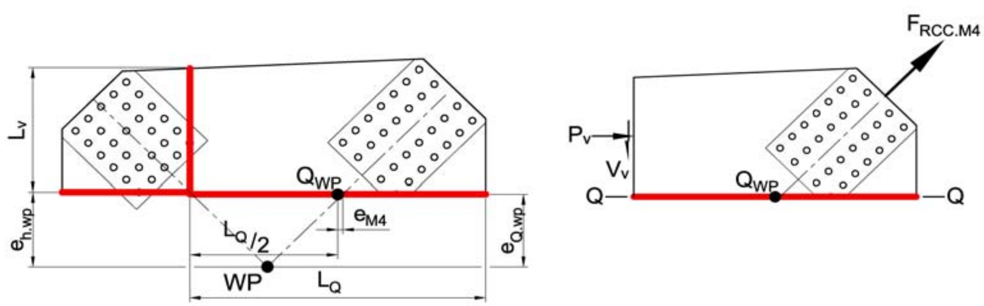


Figure 12: Remaining Gusset Plate Stub

G2.2.3c Remaining Portion (Stub) Check Cont.: Parallel Resultants

Calculate forces P_Q and V_Q along Section Q

$$P_Q = F_{RCC.M4} \cdot \sin(\theta_{M4}) - V_v = 398\text{kip} \cdot \sin(45\text{deg}) - 140\text{kip} = 142\text{kip}$$

$$V_Q = F_{RCC.M4} \cdot \cos(\theta_{M4}) + P_v = 398\text{kip} \cdot \cos(45\text{deg}) + 140\text{kip} = 421\text{kip}$$

Calculate moment M_Q acting at Q_{WP}

$$M_Q = P_v \cdot \left(\frac{L_v}{2} + e_{h.wp} - e_{Q.wp}\right) - V_v \cdot \frac{L_Q}{2} - F_{RCC.M4} \cdot \sin(\theta_{M4}) \cdot e_{M4}$$

$$M_Q = 140\text{kip} \cdot \left(\frac{18.0\text{in}}{2} + 10.8\text{in} - 10.5\text{in}\right) - 140\text{kip} \cdot \frac{41.2\text{in}}{2} - 398\text{kip} \cdot \sin(45\text{deg}) \cdot 0.7\text{in} = -1790\text{kip} \cdot \text{in}$$

Determine section modulus and calculate bending and normal stresses

$$S = \frac{{L_Q}^2 \cdot t}{6} = \frac{(41.2\text{in})^2 \cdot 0.4375\text{in}}{6} = 124\text{in}^3$$

$$\sigma_P = \frac{P_Q}{L_Q \cdot t} = \frac{142\text{kip}}{41.2\text{in} \cdot 0.4375\text{in}} = 7.86\text{ksi}$$

$$\sigma_M = \frac{|M_Q|}{S} = \frac{|-1790\text{kip} \cdot \text{in}|}{124\text{in}^3} = 14.4\text{ksi}$$

Since $\sigma_P + \sigma_M < F_y$ and $\sigma_M > \sigma_P$, use σ in von Mises equation based on $0.6*\sigma_{max}$ (Refer to Appendix A)

$$\sigma_{0.6} = 0.6 \cdot (\sigma_P + \sigma_M) = 0.6 \cdot (7.86\text{ksi} + 14.4\text{ksi}) = 13.4\text{ksi}$$

$$\Omega = \sqrt{1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2} = \sqrt{1 - \left(\frac{13.4\text{ksi}}{36.4\text{ksi}}\right)^2} = 0.93$$

$$\tau_N = \Omega \cdot (0.58) \cdot F_y = 0.93 \cdot (0.58) \cdot 36.4\text{ksi} = 19.6\text{ksi}$$

Check shear on Section Q to see if it is less than 19.6 ksi

$$v_Q = \frac{V_Q}{L_Q \cdot t} = \frac{422\text{kip}}{41.2\text{in} \cdot 0.4375\text{in}} = 23.4\text{ksi} \qquad \geq \qquad \tau_N = 19.6\,\text{ksi}$$

Therefore, remaining portion of gusset plate is overstressed based on the demands of the Refined Corner Check. Reduce input forces by the ratio of the overstress as a starting point (can refine if necessary to increase calculated capacity).

G2.2.3c Remaining Portion (Stub) Check Cont.: Parallel Resultants

$$\text{Ratio} = \frac{\tau_N}{v_Q} = \frac{19.6\text{ksi}}{23.4\text{ksi}} = 84.0\%$$

$$P_h = P_h \cdot \text{Ratio} = 142\text{kip} \cdot 84.0\% = 119\text{kip}$$

$$V_h = V_h \cdot \text{Ratio} = 142\text{kip} \cdot 84.0\% = 119\text{kip}$$

$$P_v = P_v \cdot \text{Ratio} = 140\text{kip} \cdot 84.0\% = 117\text{kip}$$

$$V_v = V_v \cdot \text{Ratio} = 140\text{kip} \cdot 84.0\% = 117\text{kip}$$

Similarly reduce stresses by the ratio of the overstress

$$\sigma_P = \sigma_P \cdot \text{Ratio} = 7.86\text{ksi} \cdot 84.0\% = 6.60\text{ksi}$$

$$\sigma_M = \sigma_M \cdot \text{Ratio} = 14.4\text{ksi} \cdot 84.0\% = 12.1\text{ksi}$$

Since $\sigma_P + \sigma_M < F_y$ and $\sigma_M > \sigma_P$, use σ in von Mises equation based on $0.6 * \sigma_{max}$ (Refer to Appendix A)

$$\sigma_{0.6} = 0.6 \cdot (\sigma_P + \sigma_M) = 0.6 \cdot (6.60\text{ksi} + 12.1\text{ksi}) = 11.2\text{ksi}$$

$$\Omega = \sqrt{1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2} = \sqrt{1 - \left(\frac{11.2\text{ksi}}{36.4\text{ksi}}\right)^2} = 0.95$$

$$\tau_N = \Omega \cdot (0.58) \cdot F_y = 0.95 \cdot (0.58) \cdot 36.4\text{ksi} = 20.1\text{ksi}$$

Therefore, remaining portion of gusset plate is not overstressed.

$$v_Q = v_Q \cdot \text{Ratio} = 23.4\text{ksi} \cdot 84.0\% = 20.1\text{ksi} \qquad = \qquad \tau_N = 20.1\text{ksi}$$

$$C_{RCC} = \sqrt{(V_h + P_v)^2 + (V_v + P_h)^2} = \sqrt{(119\text{kip} + 117\text{kip})^2 + (117\text{kip} + 119\text{kip})^2} = 334\text{kip}$$

RCC Parallel Resultants Capacity (per plate)

Total member capacity
$2 \cdot 334\text{kip} = 669\text{kip}$

G2.2.3d RCC Buckling Check: Parallel Resultants

Check buckling due to axial forces on corner surfaces with Refined Corner Check demands (refer to Appendix B)

G2.2.3d1 Short Span Buckling Check:

For this gusset plate, the short span corresponds to the horizontal surface

$F_{cr} = 27.6\text{ksi}$ See Basic Corner Check

$\sigma = \sigma_h \cdot \text{Ratio} = 18.2\text{ksi} \cdot 84.0\% = 15.3\text{ksi}$

$\tau = \tau_h \cdot \text{Ratio} = 18.2\text{ksi} \cdot 84.0\% = 15.3\text{ksi}$

$$\sigma_{Princ} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{15.3\text{ksi}}{2} + \sqrt{\left(\frac{15.3\text{ksi}}{2}\right)^2 + (15.3\text{ksi})^2} = 24.7\text{ksi} \qquad \le \qquad F_{cr} = 27.6\text{ksi}$$

The principle stress is less than the critical buckling stress; therefore, short span buckling is not a concern.

G2.2.3d2 Long Span Buckling Check:

Treat as flat rectangular plate with one non-loaded edge fixed and the remaining edges clamped

$F_{cr} = 26.3\text{ksi}$ See Basic Corner Check

$\sigma = \sigma_v \cdot \text{Ratio} = 17.8\text{ksi} \cdot 84.0\% = 14.9\text{ksi}$

$\tau = \tau_v \cdot \text{Ratio} = 17.8\text{ksi} \cdot 84.0\% = 14.9\text{ksi}$

$$\sigma_{Principle} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{14.9\text{ksi}}{2} + \sqrt{\left(\frac{14.9\text{ksi}}{2}\right)^2 + (14.9\text{ksi})^2} = 24.1\text{ksi} \qquad \le \qquad F_{cr} = 26.3\text{ksi}$$

The principle stress is less than the critical buckling stress, therefore long span buckling is not a concern

Since buckling is not a concern for the Refined Corner Check, no reduction in calculated capacity is required

$C_{RCC} = 334\text{ kip}$

RCC Parallel Resultants Capacity (per plate)

Total member capacity
$2 \cdot 334\text{kip} = 669\text{kip}$

$$ORF_{RCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{334\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -275\text{kip}\right|}{1.3 \cdot \left|-275\text{kip}\right|} = 0.87$$

$$IRF_{RCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{334\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -275\text{kip}\right|}{2.17 \cdot \left|-275\text{kip}\right|} = 0.52$$

G2.2.4 Refined Corner Check: Nonparallel Resultants:

Removing the constraint that the corner surface resultants remain parallel can result in further optimization of the shear and normal forces on the surfaces and an increase in capacity. Knowing that the necessary increase in capacity is small to have Horizontal Shear control and that the "Stub" is nearly at capacity, choose first iteration forces to avoid this overstress while increasing the overall capacity. A decrease in P_v (to decrease the 3x shear stress term along the stub) and an increase in V_v (to increase capacity) will require appropriate changes to P_h and V_h in order to have the final resultant along the member and to balance the moments.

Allowing the surface resultants to be nonparallel creates multiple equations with multiple unknowns, requiring a complex iterative approach to achive a solution. In selecting trial values for V and P, recognize that adjustments in shear have a 3x effect on shear stress when considering von Mises stress.

Considering that the stub controlled overall and the horizontal surface controlled over the vertical surface for the first iteration, try to maximize the stress on the horizontal surface. An increase in V_h will increase the capacity and decrease the shear on the stub (caused by decreasing P_v). Therefore, determine an increased V_h such that the final capacity is larger than the Horizontal Shear capacity.

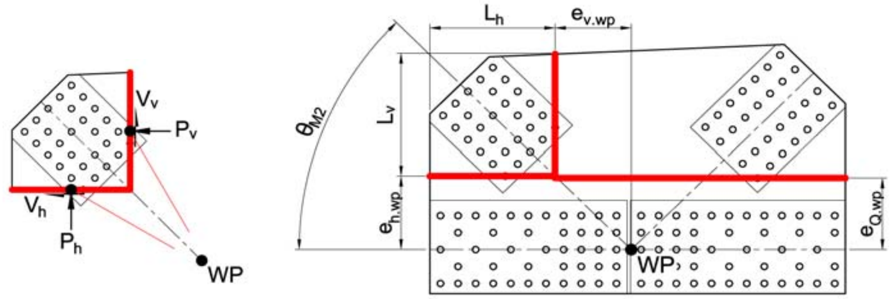


Figure 13: Refined Corner Check for Diagonal Member M2

G2.2.4a Determine Trial Forces and Overall Capacity with All Forces a Function of V_h:

G2.2.4a1 - Horizontal Surface:

Solve the von Mises stress relationship for the axial force on the horizontal surface so that P_h is a function of V_h

$$F_y^2 = \sigma^2 + 3\cdot\tau^2$$

$$F_y^2 = \left(\frac{P_h}{L_h\cdot t}\right)^2 + 3\cdot\left(\frac{V_h}{L_h\cdot t}\right)^2$$

$$P_h = \sqrt{F_y^2\cdot L_h^2\cdot t^2 - 3\cdot V_h^2}$$

G2.2.4a2 - Vertical Surface:

Solve for the forces acting on the vertical surface as a function of the forces acting on the horizontal surface

Constrain final resultant to be parallel to member to avoid bending in member

$$\operatorname{atan}\left(\frac{P_h + V_v}{V_h + P_v}\right) = \theta_{M2}$$

Constrain moments about work point to balance

$$P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) - V_h \cdot e_{h.wp} - P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right) + V_v \cdot e_{v.wp} = 0$$

Solve two equations for P_h and V_h

$$P_v = \frac{P_h + V_v}{\tan(\theta_{M2})} - V_h$$

$$V_v = \frac{P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right) + V_h \cdot e_{h.wp} - P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right)}{e_{v.wp}}$$

$$P_v = \frac{P_h + \frac{P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right) + V_h \cdot e_{h.wp} - P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right)}{e_{v.wp}}}{\tan(\theta_{M2})} - V_h$$

Substitute for P_v and V_v combine terms and simplify

$$P_v = \frac{L_h \cdot P_h - 2 \cdot V_h \cdot e_{h.wp} + 2 \cdot V_h \cdot e_{v.wp} \cdot \tan(\theta_{M2})}{L_v + 2 \cdot e_{h.wp} - 2 \cdot e_{v.wp} \cdot \tan(\theta_{M2})}$$

G2.2.4a3 - Trial Force Substitution:

Choose a value for the shear on the horizontal surface (V_h) that gives a calculated capacity just above that of Horizontal Shear.

Recall: $C_{HS.M2} = 343\,kip$ Therefore, select $V_h = 118.95\,kip$

$$P_h = \sqrt{F_y^2 \cdot L_h^2 \cdot t^2 - 3 \cdot V_h^2} = \sqrt{(36.4ksi)^2 \cdot (17.8in)^2 \cdot (0.4375in)^2 - 3 \cdot (150kip)^2} = 113kip$$

$$P_v = \frac{17.8in \cdot 113kip - 2 \cdot 150kip \cdot 10.8in + 2 \cdot 150kip \cdot 10.8in \cdot \tan(45deg)}{18in + 2 \cdot 10.8in - 2 \cdot 10.8in \cdot \tan(45deg)} = 112kip$$

$$V_v = \frac{112kip \cdot \left(\frac{18in}{2} + 10.8in\right) + 150kip \cdot 10.8in - 113\,kip\left(\frac{17.8in}{2} + 10.8in\right)}{10.8in} = 149kip$$

$$C_{RCC} = \sqrt{\left(V_h + P_v\right)^2 + \left(V_v + P_h\right)^2} = \sqrt{(150kip + 112kip)^2 + (149kip + 113kip)^2} = 370kip$$

Considering the capacity would be nearly 10% larger (if stress checks are OK) than that of Horizontal Shear, consider decreasing trial forces (and therefore capacity) prior to performing stress checks. The stress checks will be more likely to work out, reducing the number of iterations. Note that increasing V_h, in this range, decreases capacity and decreases shear on the stub.

$V_h = 150\,kip$

$$P_h = \sqrt{F_y^2 \cdot L_h^2 \cdot t^2 - 3 \cdot V_h^2} = \sqrt{(36.4ksi)^2 \cdot (17.8in)^2 \cdot (0.4375in)^2 - 3 \cdot (155kip)^2} = 90kip$$

$$P_v = \frac{17.8in \cdot 90kip - 2 \cdot 155kip \cdot 10.8in + 2 \cdot 155kip \cdot 10.8in \cdot \tan(45deg)}{18in + 2 \cdot 10.8in - 2 \cdot 10.8in \cdot \tan(45deg)} = 89kip$$

$$V_v = \frac{89kip \cdot \left(\frac{18in}{2} + 10.8in\right) + 155kip \cdot 10.8in - 90kip \cdot \left(\frac{17.8in}{2} + 10.8in\right)}{10.8in} = 154kip$$

$$C_{RCC} = \sqrt{\left(V_h + P_v\right)^2 + \left(V_v + P_h\right)^2} = \sqrt{(155kip + 89kip)^2 + (154kip + 90kip)^2} = 346kip$$

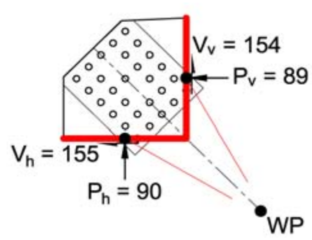


Figure 14: Refined Corner Check Resultants with Resultants not Parallel to Member

If the stress checks are adequate, this combination of forces would give a capacity just greater than that calculated by Horizontal Shear. Proceed knowing that the horizontal surface already is at maximum capacity and does not need to be checked.

G.2.2.4b Vertical Surface Check: Nonparallel Resultants

Constrain moments about work point to balance

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{89\text{kip}}{(18.0\text{in})\cdot(0.4375\text{in})} = 11.3\text{ksi} \qquad \tau_v = \frac{V_v}{L_v \cdot t} = \frac{154\text{kip}}{(18.0\text{in})\cdot(0.4375\text{in})} = 19.5\text{ksi}$$

$$\sigma_{vm.v} = \sqrt{\sigma_v^{\ 2} + 3\tau_v^{\ 2}} = \sqrt{(11.3\text{ksi})^2 + 3\cdot(19.5\text{ksi})^2} = 35.7\text{ksi} \quad \le \quad F_y = 36.4\text{ksi}$$

G.2.2.4c Remaining Portion (Stub) Check: Nonparallel Resultants

Calculate equivalent concurrent forces for tension diagonal

$$F_{RCC.M4} = \text{OpForce}_{M4}\cdot\left|\frac{C_{RCC}}{\text{OpForce}_{M2}}\right| = 716\text{kip}\cdot\left|\frac{346\text{kip}}{-716\text{kip}}\right| = 346\text{kip}$$

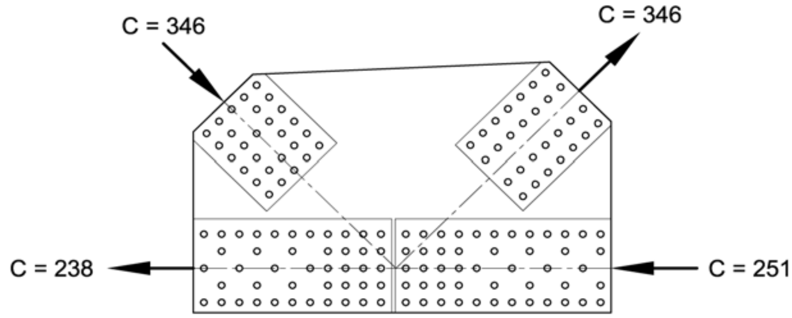


Figure 15: Concurrent Member Capacities (per plate) Based on Refined Corner Check (Subject to Stub Check and Buckling Check)

Check remaining portion of the gusset plate outside of the corner and chord. Select a Section Q that encompasses all forces applied by member M4.

$L_Q = 41.2\text{in}$

$e_{Q.wp} = 10.5\text{ in}$

$\theta_{M4} = 45\cdot\text{deg}$

$e_{M4} = 0.7\text{ in}$

$L_v = 18\text{ in}$

$e_{h.wp} = 10.8\text{ in}$

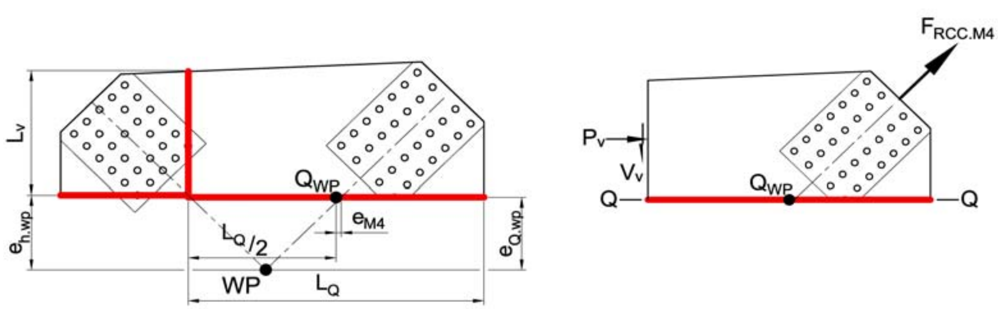


Figure 12: Remaining Gusset Plate Stub

G2.2.4c Stub Check Cont.: Nonparallel Resultants

Calculate forces P_Q and V_Q along Section Q

$$P_Q = F_{RCC.M4} \cdot \sin(\theta_{M4}) - V_v = 346kip \cdot \sin(45deg) - 154kip = 90kip$$

$$V_Q = F_{RCC.M4} \cdot \cos(\theta_{M4}) + P_v = 346kip \cdot \cos(45deg) + 89kip = 334kip$$

Calculate moment M_Q about Section Q

$$M_Q = P_v \cdot \left(\frac{L_v}{2} + e_{h.wp} - e_{Q.wp}\right) - V_v \cdot \frac{L_Q}{2} - F_{RCC.M4} \cdot \sin(\theta_{M4}) \cdot e_{M4}$$

$$M_Q = 89kip \cdot \left(\frac{18.0in}{2} + 10.8in - 10.5in\right) - 154kip \cdot \frac{41.2in}{2} - 346kip \cdot \sin(45deg) \cdot 0.7in = -2510kip \cdot in$$

Determine section modulus and calculate bending and normal stresses

$$S = \frac{L_Q^{\,2} \cdot t}{6} = \frac{(41.2in)^2 \cdot \frac{7}{16}in}{6} = 124in^3$$

$$\sigma_P = \frac{P_Q}{L_Q \cdot t} = \frac{90kip}{41.2in \cdot \frac{7}{16}in} = 5.0ksi$$

$$\sigma_M = \frac{|M_Q|}{S} = \frac{|-2510kip \cdot in|}{124in^3} = 20.3ksi$$

Since $\sigma_P + \sigma_M < F_y$ and $\sigma_M > \sigma_P$, use σ in von Mises equation based on $0.6*\sigma_{max}$ (Refer to Appendix A)

$$\sigma_{0.6} = 0.6 \cdot (\sigma_P + \sigma_M) = 0.6 \cdot (5.0ksi + 20.3ksi) = 15.2ksi$$

$$\Omega = \sqrt{1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2} = \sqrt{1 - \left(\frac{15.2ksi}{36.4ksi}\right)^2} = 0.91$$

$$\tau_N = \Omega \cdot (0.58) \cdot F_y = 0.91 \cdot (0.58) \cdot 36.4ksi = 19.2ksi$$

Check shear on Section Q to see if it is less than 19.2 ksi

$$v_Q = \frac{V_Q}{L_Q \cdot t} = \frac{334kip}{41.2in \cdot \frac{7}{16}in} = 18.5ksi \qquad \leq \qquad \tau_N = 19.2\,ksi$$

Therefore, remaining portion of gusset plate can sustain the demands of the Refined Corner Check.

G2.2.4d Buckling Check: Nonparallel Resultants

Check buckling due to axial forces on surfaces (refer to Appendix B)

G2.2.4d1 Short Span Buckling Check:

For this gusset plate, the short span corresponds to the horizontal surface

$F_{cr} = 27.6\text{ksi}$ See Basic Corner Check

$$\sigma = \sigma_h = \frac{P_h}{L_h \cdot t} = \frac{90\text{kip}}{(17.8\text{in}) \cdot \left(\frac{7}{16}\text{in}\right)} = 11.6\text{ksi} \qquad \tau = \tau_h = \frac{V_h}{L_h \cdot t} = \frac{155\text{kip}}{(17.8\text{in}) \cdot \left(\frac{7}{16}\text{in}\right)} = 19.9\text{ksi}$$

$$\sigma_{Principle} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{11.6\text{ksi}}{2} + \sqrt{\left(\frac{11.6\text{ksi}}{2}\right)^2 + (19.9\text{ksi})^2} = 26.6\text{ksi} \qquad \le \qquad F_{cr} = 27.6\text{ksi}$$

The principle stress is less than the critical buckling stress, therefore short span buckling is not a concern

G2.2.4d2 Long Span Buckling Check:

Treat as flat rectangular plate with one non loaded edge fixed and the remaining edges clamped

$F_{cr} = 26.3\text{ksi}$ See Basic Corner Check

$\sigma = \sigma_v = 11.3\text{ksi}$

$\tau = \tau_v = 19.5\text{ksi}$

$$\sigma_{Principle} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{11.3\text{ksi}}{2} + \sqrt{\left(\frac{11.3\text{ksi}}{2}\right)^2 + (19.5\text{ksi})^2} = 26.0\text{ksi} \qquad \le \qquad F_{cr} = 26.3\text{ksi}$$

The principle stress is less than the critical buckling stress, therefore long span buckling is not a concern

Since buckling was not a concern for the Basic Corner Check, no reduction in calculated capacity is required

RCC Nonparallel Resultants Capacity (per plate)

$C_{RCC} = 346\,\text{kip}$

Total member capacity
$2 \cdot 346\text{kip} = 691\text{kip}$

$$ORF_{RCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{346\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -275\text{kip}\right|}{1.3 \cdot |-275\text{kip}|} = 0.93$$

$$IRF_{RCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{346\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -275\text{kip}\right|}{2.17 \cdot |-275\text{kip}|} = 0.56$$

Because this result for the Refined Corner Check is greater than result from Horizontal Shear, no further iterations are necessary.

G2.2.5 Evaluation Summary:

The Refined Corner Check replaces the Basic Corner Check when determining capacities. Since the calculations indicate that the capacity of this gusset plate (based on the compression diagonal) is now controlled by Horizontal Shear (and not Refined Corner Check) there is no benefit from further refinement. This increases the calculated capacity from 300 kips to 343 kips (14%).

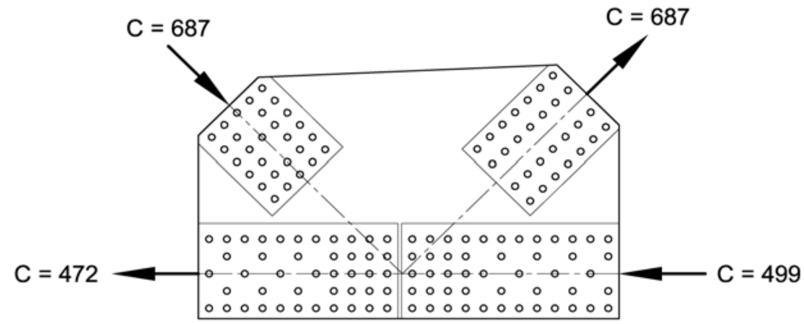


Figure 17: Concurrent Member Capacities Based on Refined Analysis (for Gusset Plate Pair)

Limit State	Gusset Plate Pair	
	Operating Rating	Inventory Rating
Fasteners	3.05	1.83
Vertical Shear	2.45	1.47
~~Horizontal Shear~~[1]	~~0.89~~	~~0.54~~
~~Partial Shear Yield~~[2]	~~0.68~~	~~0.41~~
~~Whitmore Compression~~[2]	~~0.69~~	~~0.41~~
Tension	1.48	0.89
Block Shear	1.43	0.85
Chord Splice	107.0	64.3
Horizontal Shear (Ω Calc.)	0.92	0.55
~~Basic Corner Check~~[3]	0.90	0.54
Refined Corner Check	0.93	0.56

Controls

[1] Superceded by Horizontal Shear with Ω calculated.

[2] Superceded by Basic Corner Check (see [3]).

[3] Superceded by final iteration of Refined Corner Check.

By refining the analysis calculations using the approach presented above, a nearly 35% increase in the Operating Rating can be achieved. Although the Operating Rating of the gusset plate is still below 1.0, the type of repair necessary to increase the gusset plate capacity is better understood which should lead to a more effiecient repair. In the meantime, bridge posting limits prior to installation of repairs would be less restrictive due to the refined analysis.

Example 3 - Noncompact Gusset Plate with Medium Buckling Length between Diagonals

Example 3 is a four member gusset plate (no vertical) with a medium buckling length between diagonals. It is not a compact gusset plate and no members are chamfered. Calculations apply to one of the two gusset plates.

G3.1 Gusset Plate Material, Geometric, and Loading Properties:

Material Properties

$F_y = 53\text{ksi}$

$F_u = 80\text{ksi}$

$E = 29000\text{ksi}$

$\nu = 0.3$

Plate Thickness

$t = \frac{1}{2}\text{in}$

Member Angles

$\theta_{M2} = 50.47\text{deg}$

$\theta_{M3} = 68.23\text{deg}$

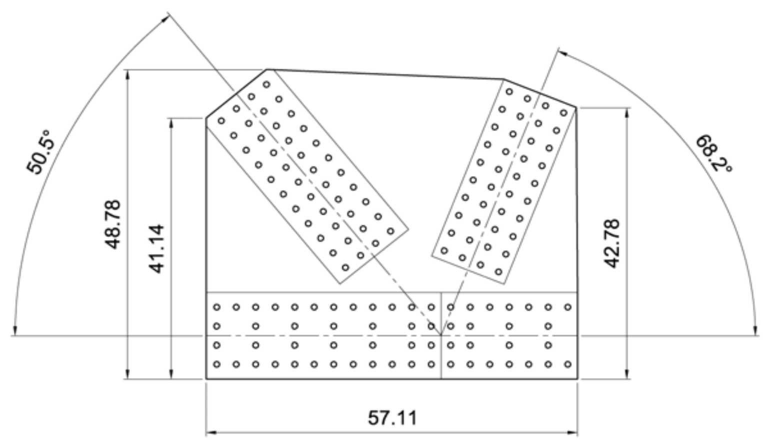


Figure 1: Basic Geometry of Gusset Plate

Unfactored Member Forces Per Gusset Plate Pair

$LL_{M1} = 112\,\text{kip}$

$DL_{M1} = 449\,\text{kip}$

$LL_{M2} = -215\,\text{kip}$

$DL_{M2} = -862\,\text{kip}$

$LL_{M3} = 183\,\text{kip}$

$DL_{M3} = 732\,\text{kip}$

$LL_{M5} = -91\,\text{kip}$

$DL_{M5} = -363\,\text{kip}$

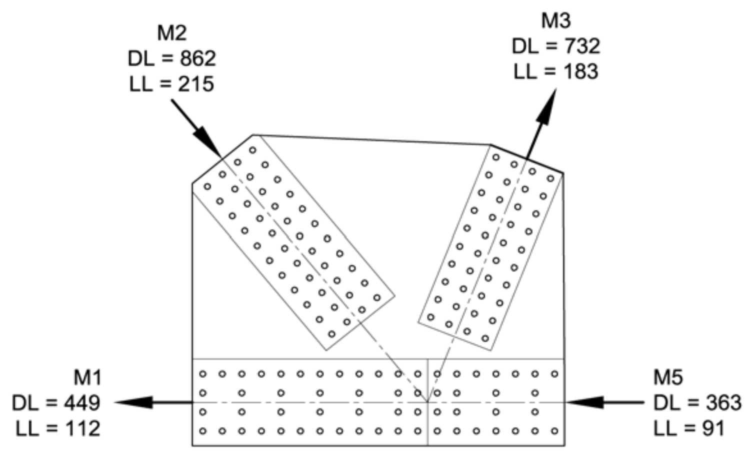


Figure 2: Concurrent Member Forces Transferred to Two Gusset Plates

Member forces based on NCHRP Project 12-84 loads with an assumed Dead Load to Live Load ratio of 80/20.

G3.1 Gusset Plate Material, Geometric, and Loading Properties Cont.:

Factored Forces Acting on Gusset Plate Pair

$InvForce_{M1} = \gamma_{InvLL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 2.17 \cdot 112kip + 1.3 \cdot 449kip$ $\quad InvForce_{M1} = 828\,kip$

$OpForce_{M1} = \gamma_{LL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 1.3 \cdot 112kip + 1.3 \cdot 449kip$ $\quad OpForce_{M1} = 730\,kip$

$InvForce_{M2} = -1587\,kip$

$OpForce_{M2} = -1400\,kip$

$InvForce_{M3} = 1349\,kip$

$OpForce_{M3} = 1190\,kip$

$InvForce_{M5} = -669\,kip$

$OpForce_{M5} = -590\,kip$

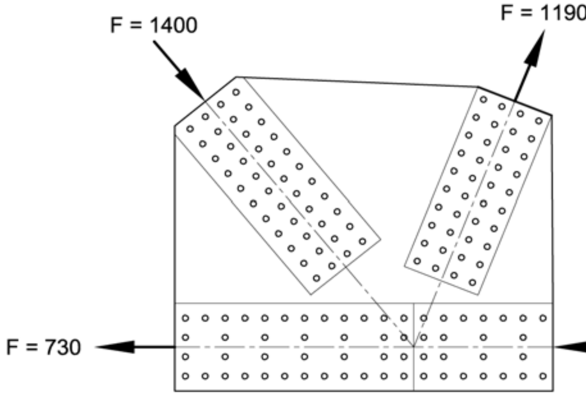


Figure 3: Concurrent Member Operating Forces Transferred to Two Gusset Plates

G3.2 Evaluation Approach:

In accordance with the *2014 Interim Revisions to the Manual for Bridge Evaluation, Second Edition,* the following gusset plate limit state checks were done:
(a) Fastener strength (L6B.2.6.1)
(b) Vertical shear resistance (L6B.2.6.3)
(c) Horizontal shear resistance (L6B.2.6.3)
(d) Partial shear yield resistance (L6B.2.6.3)
(e) Compressive (Whitmore) resistance (L6B.2.6.4)
(f) Tension strength (L6B.2.6.5)
(g) Bock shear resistance (L6B.2.6.5)
(h) Chord splice capacity (L6B.2.6.6)

Load Factor Rating Summary for Example 3

Limit State	Gusset Plate Pair		
	Operating Rating	Inventory Rating	
Fasteners	2.62	1.57	7/8 in. diam A502 Grade I fasteners
Vertical Shear	3.60	2.16	Ω=0.88 with splice plates included
Horizontal Shear	1.69	1.01	
Partial Shear Yield	1.02	0.61	**Controls**
Whitmore Compression	2.48	1.48	
Tension	4.06	2.43	
Block Shear	3.99	2.39	
Chord Splice	45.9	27.5	

When the Partial Shear Plane Yield and/or Whitmore Compression capacity checks control and indicate a less than acceptable rating, more rigorous evaluation should be performed.

The following more rigorous rating checks are performed in Example 3:
(1) Horizontal shear capacity - Ω calculated: Supercedes Horizontal Shear with Ω = 0.88.
(2) Basic Corner Check capacity (BCC): Replaces Partial Shear Plane Yield and Whitmore Compression capacity checks.

G3.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω):

Global shear check along horizontal plane parallel with bottom chord. Shear force calculated using horizontal component of diagonal member forces. Gross section selected at bottom fastener of diagonal members to achieve maximum eccentricity. Net section calculated through bottom chord fastener holes. Ω calculated using Drucker formula.

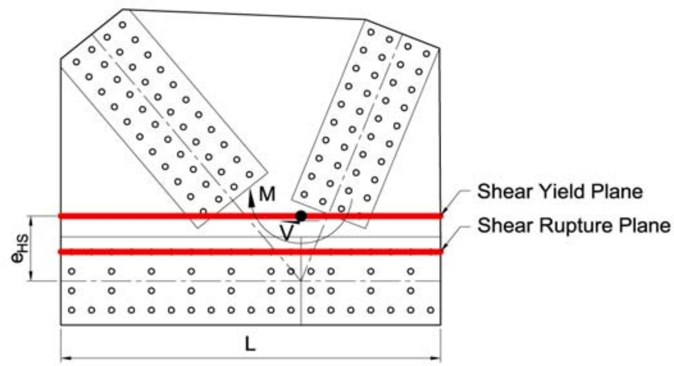


Figure 4: Horizontal Shear Between Web and Chord Members

$L = 57.1\,in$

$e_{HS} = 10.1\,in$

$M = V \cdot e_{HS}$

$A_g = t \cdot L_{HS} = \frac{1}{2} in \cdot 57.1in = 28.6in^2$

$d_h = 1\,in$

$n_{hole} = 19$

$A_n = t \cdot \left(L - n_{hole} \cdot d_h\right) = \frac{1}{2} in \cdot [57.1in - (19) \cdot 1.0in] = 19.1in^2$

Calculate Ω using Drucker formula instead of using AASHTO-specified Ω=0.88

$$V = V_p \cdot \left[1 - \left(\frac{M}{M_p}\right)\right]^{0.25} \qquad V = \Omega \cdot V_p$$

Drucker Formula [1]

$V_P = (0.58) \cdot F_y \cdot A_g = (0.58) \cdot 53ksi \cdot 28.6in^2 = 878kip$

$$M_P = \frac{L^2 \cdot t}{4} \cdot F_y = \frac{(57.1in)^2 \cdot \frac{1}{2} in}{4} \cdot 53ksi = 21600in \cdot kip$$

Substitute $V = \Omega * V_p$ into Drucker formula and rearrange to solve for Ω using plastic shear and moment capacities

$$\Omega \cdot V_p = V_p \cdot \left(1 - \frac{\Omega \cdot V_p \cdot e_{HS}}{M_p}\right)^{0.25}$$

$$\Omega = \left(1 - \frac{\Omega \cdot V_P \cdot e_{HS}}{M_P}\right)^{0.25} = \left(1 - \frac{\Omega \cdot 623 \cdot 878 \cdot 10.1in}{21600in \cdot kip}\right)^{0.25} = 0.89$$

Requires iterative process since V is proportional to Ω. Can substitute AASHTO specified value of Ω = 0.88 on right side of equation as a first estimate of Ω. Result shown is the calculated value of Ω after performing necessary iterations.

G3.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω) Cont.:

$\Phi_{vy} = 1.0$

$\Phi_{vu} = 0.85$

$C_Y = \varphi_{yy}\cdot(0.58)\cdot F_y\cdot A_g\cdot\Omega = 1.00(0.58)\cdot 53\text{ksi}\cdot 28.6\text{in}^2\cdot(0.89) = 784\text{kip}$

$C_U = \varphi_{yu}\cdot(0.58)\cdot F_u\cdot A_n = 0.85(0.58)\cdot 80\text{ksi}\cdot 19.1\text{in}^2 = 752\text{kip}$

$C_{HS} = \min(C_Y, C_U) = \min(784\text{kip}, 752\text{kip}) = 752\text{kip}$

Horizontal Shear Capacity (per plate)

Note that the horizontal shear capacity is controlled by shear rupture and is not dependent on Ω being 0.88 or calculated.

Determine capacity of member M2 based on Horizontal Shear

$$C_{HS.M2} = C_{HS}\cdot\left|\frac{OpForce_{M2}}{OpForce_{M1} - OpForce_{M5}}\right| = 752\text{kip}\cdot\left|\frac{-1400\text{kip}}{730\text{kip} - (-590\text{kip})}\right| = 797\text{kip}$$

Total member capacity
$2\cdot 797\text{kip} = 1594\text{kip}$

$$ORF_{HS} = \frac{C_{HS.M2} - \gamma_{DL}\cdot\left|\frac{1}{2}DL_{M2}\right|}{\gamma_{LL}\cdot\left|\frac{1}{2}LL_{M2}\right|} = \frac{797\text{kip} - 1.3\cdot\left|\frac{1}{2}\cdot -862\text{kip}\right|}{1.3\cdot\left|\frac{1}{2}\cdot -215\text{kip}\right|} = 1.69$$

$$IRF_{HS} = \frac{C_{HS.M2} - \gamma_{DL}\cdot\left|\frac{1}{2}DL_{M2}\right|}{\gamma_{InvLL}\cdot\left|\frac{1}{2}LL_{M2}\right|} = \frac{797\text{kip} - 1.3\cdot\left|\frac{1}{2}\cdot -862\text{kip}\right|}{2.17\cdot\left|\frac{1}{2}\cdot -215\text{kip}\right|} = 1.01$$

[1] Drucker, D., *The Effect of Shear on the Plastic Bending of Beams*, American Society of Mechanical Engineers, NAMD Conference, Urbana, IL, June 1956

G3.2.2 Basic Corner Check:

The Basic Corner Check is a first-principles analytical approach utilizing fundamental steel design theory to conservatively calculate gusset plate limit state capacities at critical cross sections. This check is used to evaluate equilibrium and stability of a gusset plate "corner" bounded by horizontal and vertical planes that create the smallest section encompassing all fasteners of the diagonal member. The diagonal member force is assumed to be resisted by a combination of shear and normal forces acting on the vertical and horizontal surfaces bounding the "corner". Von Mises stress calculated on the surfaces is limited to the yield strength of the gusset plate. For simplicity and to avoid bending in the members, the resultant of each surface must pass through the work point. The "corner" can be adjusted in terms of location and plate thickness to accommodate deterioration.

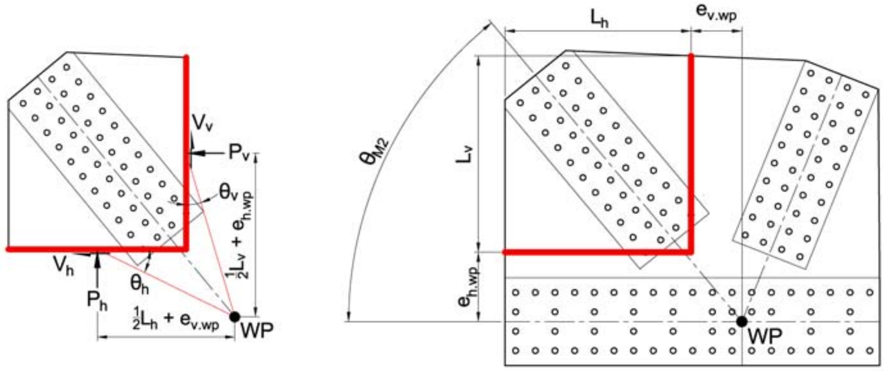


Figure 5: Basic Corner Check for Diagonal M2 Member

Calculate resultant angles from the work point

$L_h = 28.4\,\text{in}$ $\quad e_{h.wp} = 10.7\,\text{in}$

$L_v = 30.4\,\text{in}$ $\quad e_{v.wp} = 7.75\,\text{in}$

$$\theta_h = \operatorname{atan}\left(\frac{e_{h.wp}}{\frac{L_h}{2} + e_{v.wp}}\right) = \operatorname{atan}\left(\frac{10.7\text{in}}{\frac{28.4\text{in}}{2} + 7.75\text{in}}\right) = 26.1\text{deg}$$

$$\theta_v = \operatorname{atan}\left(\frac{e_{v.wp}}{\frac{L_v}{2} + e_{h.wp}}\right) = \operatorname{atan}\left(\frac{7.75\text{in}}{\frac{30.4\text{in}}{2} + 10.7\text{in}}\right) = 16.6\text{deg}$$

G3.2.2a Horizontal Surface Check:

Since $L_h < L_v$ set von Mises stress on horizontal surface equal to plate yield strength. After stresses on both surfaces are determined, verify assumption that horizontal surface is critical (i.e. reaches von Mises yield before vertical surface).

$$P_h = V_h \cdot \tan(\theta_h)$$

$$\sigma_h = \frac{P_h}{A_h} = \frac{P_h}{L_h \cdot t}$$

$$\tau_h = \frac{V_h}{A_h} = \frac{V_h}{L_h \cdot t}$$

$$\sigma_{vm} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}}$$

Substitute P_h as a function of V_h and set the von Mises stress to yield

$$F_y = 53\text{ksi} = \sigma_{vm} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}} = \sqrt{\left(\frac{P_h}{L_h \cdot t}\right)^2 + 3\cdot\left(\frac{V_h}{L_h \cdot t}\right)^2} = \sqrt{\left(\frac{V_h \cdot \tan(\theta_h)}{L_h \cdot t}\right)^2 + 3\cdot\left(\frac{V_h}{L_h \cdot t}\right)^2}$$

Rearrange terms and solve for V_h

$$V_h = \frac{F_y \cdot L_h \cdot t}{\sqrt{\tan(\theta_h)^2 + 3}} = \frac{53\text{ksi} \cdot 28.4\text{in} \cdot \frac{1}{2}\text{in}}{\sqrt{\tan(26.1\text{deg})^2 + 3}} = 418\text{kip}$$

Solve for P_h

$$P_h = V_h \cdot \tan(\theta_h) = 418\text{kip} \cdot \tan(26.1\text{deg}) = 204\text{kip}$$

Calculate shear and normal stresses on horizontal surface

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{204\text{kip}}{(28.4\text{in}) \cdot \left(\frac{1}{2}\text{in}\right)} = 14.4\text{ksi} \qquad \tau_h = \frac{V_h}{L_h \cdot t} = \frac{418\text{kip}}{(28.4\text{in}) \cdot \left(\frac{1}{2}\text{in}\right)} = 29.5\text{ksi}$$

G3.2.2b Vertical Surface Check:

Determine forces and stresses on vertical surface based on horizontal surface forces and stated constraints (i.e. force resultants to pass thru workpoint).

$$P_v = V_v \cdot \tan(\theta_v)$$

$$\theta_v = 16.6 \cdot \text{deg}$$

Constrain final resultant to act along member

$$\theta_{M2} = \text{atan}\left(\frac{V_v + P_h}{P_v + V_h}\right)$$

G3.2.2b Vertical Surface Check Cont.:

$$\theta_{M2} = \operatorname{atan}\left(\frac{V_v + P_h}{P_v + V_h}\right) = \operatorname{atan}\left(\frac{V_v + P_h}{V_v \cdot \tan(\theta_v) + V_h}\right) = \operatorname{atan}\left(\frac{V_v + 204\text{kip}}{V_v \cdot \tan(16.6\text{deg}) + 418\text{kip}}\right)$$

Rearrange terms and solve for V_v. Substitute values obtained from previously solving P_h and V_h.

$$V_v = \frac{P_h - V_h \cdot \tan(\theta_{M2})}{\tan(\theta_{M2}) \cdot \tan(\theta_v) - 1} = \frac{204\text{kip} - 418\text{kip} \cdot \tan(50.5\text{deg})}{\tan(50.5\text{deg}) \cdot \tan(16.6\text{deg}) - 1} = 473\text{kip}$$

Solve for P_v

$$P_v = V_v \cdot \tan(\theta_v) = 473\text{kip} \cdot \tan(16.6\text{deg}) = 141\text{kip}$$

Calculate shear and normal stresses on vertical surface

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{141\text{kip}}{(30.4\text{in}) \cdot \left(\frac{1}{2}\text{in}\right)} = 9.3\text{ksi} \qquad \tau_v = \frac{V_v}{L_v \cdot t} = \frac{473\text{kip}}{(30.4\text{in}) \cdot \left(\frac{1}{2}\text{in}\right)} = 31.1\text{ksi}$$

$$\sigma_{vm.v} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{(9.3\text{ksi})^2 + 3 \cdot (31.1\text{ksi})^2} = 54.6\text{ksi} \qquad \geq \qquad F_y = 53\text{ksi}$$

Since the von Mises stress on the vertical surface is greater than the yield strength of the gusset plate, the vertical surface must control. Perform second iteration of calculations while setting the von Mises stress on the vertical surface to the yield stress and then determining the necessary resultants on the horizontal surface to balance the moment about the work point.

G3.2.2b1 Second Iteration - Knowing Vertical Surface Controls:

G3.2.2b2 Determine Forces on Vertical Surface:

Knowing that the vertical surface controls this particular corner check, determine forces acting on vertical surface.

$$P_v = V_v \cdot \tan(\theta_v)$$

Substitute P_v as a function of V_v and set the von Mises stress to yield

$$F_y = 53\text{ksi} = \sigma_{vm} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{\left(\frac{P_v}{L_v \cdot t}\right)^2 + 3 \cdot \left(\frac{V_v}{L_v \cdot t}\right)^2} = \sqrt{\left(\frac{V_v \cdot \tan(16.6\text{deg})}{L_v \cdot t}\right)^2 + 3 \cdot \left(\frac{V_v}{L_v \cdot t}\right)^2}$$

Rearrange terms and solve for V_v

$$V_v = \frac{L_v \cdot F_y \cdot t}{\sqrt{\tan(\theta_v)^2 + 3}} = \frac{30.4\text{in} \cdot 53\text{ksi} \cdot \frac{1}{2}\text{in}}{\sqrt{\tan(16.6\text{deg})^2 + 3}} = 459\text{kip}$$

Solve for P_v

$$P_v = V_v \cdot \tan(\theta_v) = 459\text{kip} \cdot \tan(16.6\text{deg}) = 137\text{kip}$$

Calculate shear and normal stresses on vertical surface (to use when checking buckling strength).

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{137\text{kip}}{(30.4\text{in}) \cdot \left(\frac{1}{2}\text{in}\right)} = 9.0\text{ksi} \qquad \tau_v = \frac{V_v}{L_v \cdot t} = \frac{459\text{kip}}{(30.4\text{in}) \cdot \left(\frac{1}{2}\text{in}\right)} = 30.2\text{ksi}$$

G3.2.2b3 Determine Forces on Horizontal Surface:

Determine forces and stresses on horizontal surface based on vertical surface forces and stated constraints (i.e. force resultants to pass thru workpoint).

Check the horizontal surface:

$$P_h = V_h \cdot \tan(\theta_h)$$

Constrain final resultant to act along member and substitute P_h as a function of V_h

$$\theta_{M2} = \mathrm{atan}\left(\frac{V_v + P_h}{P_v + V_h}\right) = \mathrm{atan}\left(\frac{V_v + V_h \cdot \tan(\theta_h)}{P_v + V_h}\right) = \mathrm{atan}\left(\frac{459\mathrm{kip} + V_h \cdot \tan(26.1\mathrm{deg})}{137\mathrm{kip} + V_h}\right)$$

Rearrange terms and solve for V_h. Substitute values obtained from previously solving P_v and V_v.

$$V_h = \frac{V_v - P_v \cdot \tan(\theta_{M2})}{\tan(\theta_{M2}) - \tan(\theta_h)} = \frac{459\mathrm{kip} - 137\mathrm{kip} \cdot \tan(50.5\mathrm{deg})}{\tan(50.5\mathrm{deg}) - \tan(26.1\mathrm{deg})} = 405\mathrm{kip}$$

Solve for P_h

$$P_h = V_h \cdot \tan(\theta_h) = 405\mathrm{kip} \cdot \tan(26.1\mathrm{deg}) = 198\mathrm{kip}$$

Calculate shear and normal stresses on vertical surface

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{198\mathrm{kip}}{(28.4\mathrm{in}) \cdot \left(\frac{1}{2}\mathrm{in}\right)} = 14.0\mathrm{ksi} \qquad \tau_{hi} = \frac{V_h}{L_h \cdot t} = \frac{405\mathrm{kip}}{(28.4\mathrm{in}) \cdot \left(\frac{1}{2}\mathrm{in}\right)} = 28.6\mathrm{ksi}$$

Calculate von Mises stress

$$\sigma_{vm.h} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}} = \sqrt{(14.0\mathrm{ksi})^2 + 3 \cdot (28.6\mathrm{ksi})^2} = 51.42\mathrm{ksi} \qquad \leq \qquad F_y = 53\mathrm{ksi}$$

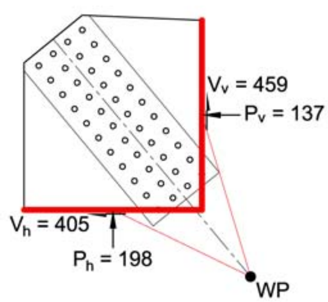


Figure 6: Basic Corner Check Resultants for Diagonal Member M2

$$C_{BCC.vM} = \sqrt{(V_h + P_v)^2 + (V_v + P_h)^2} = \sqrt{(405\mathrm{kip} + 137\mathrm{kip})^2 + (459\mathrm{kip} + 198\mathrm{kip})^2} = 852\mathrm{kip}$$

G3.2.2c BCC Buckling Check:

Check plate buckling due to axial forces on Basic Corner Check surfaces (refer to Appendix B). If buckling controls, then von Mises stresses must be adjusted.

Evaluate buckling capacity of plate regions defined by the short and long span between the Basic Corner Check surfaces and adjacent member connections.

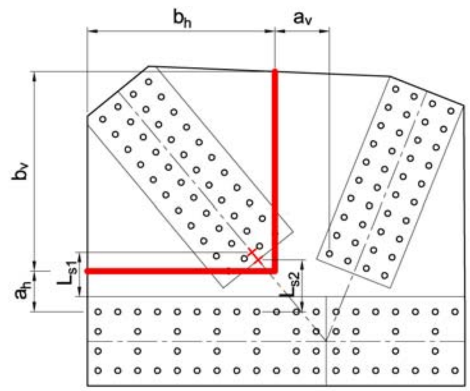


Figure 7: Corner Check Buckling Lengths

G3.2.2c1 Short Span Buckling Check:

For this gusset plate, the short span corresponds to the horizontal surface ($a_h < a_v$). a_h and a_v are defined as the distances from the respective Corner Check surface to the parallel line passing through the nearest fastener in an adjacent member.

$$L_s = \frac{L_{s1} + L_{s2}}{2} = \frac{6.8in + 7.9in}{2} = 7.4in$$

$$r = \frac{t}{\sqrt{12}} = \frac{\frac{1}{2}in}{\sqrt{12}} = 0.14in$$

Short span controls sidesway buckling, and rotation at each end is restrained. Therefore, K = 1.0 used.

$$F_e = \frac{\pi^2 \cdot E}{\left(\frac{K \cdot L_s}{r}\right)^2} = \frac{\pi^2 \cdot 29000ksi}{\left[\frac{1.0 \cdot (7.4in)}{0.14in}\right]^2} = 110ksi$$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2 \cdot \sqrt{2}}\right) = 53ksi \cdot \left(1 - \frac{\sqrt{\frac{53ksi}{110ksi}}}{2 \cdot \sqrt{2}}\right) = 40.0ksi$$

G3.2.2c1 Short Span Buckling Check Cont.:

$$\sigma = \sigma_h = 14.0\text{ksi}$$

$$\tau = \tau_h = 28.6\text{ksi}$$

$$\sigma_{Principle} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{14.0\text{ksi}}{2} + \sqrt{\left(\frac{14.0\text{ksi}}{2}\right)^2 + (28.6\text{ksi})^2} = 36.4\text{ksi} \quad \leq \quad F_{cr} = 40.0\text{ksi}$$

Principle stress is less than the critical buckling stress; therefore, buckling of short span does not control.

G3.2.2c2 Long Span Buckling Check:

Treat long span as flat rectangular plate with one non-loaded edge fixed and the remaining edges clamped (dashed curve D in Figure 8)

Long Span Length (Figure 8)

$$a = a_v = 8.25\text{in}$$

Length of Long Side Surface (Figure 8)

$$b = L_v = 30.4\text{in}$$

$$\frac{a}{b} = \frac{8.25\text{in}}{30.4\text{in}} = 0.27$$

Because a/b is less than 0.75 (where k curve is nearly asymptotic), buckling of long span plate is not a concern. Otherwise calculate k as follows (using an approximate best fit function of dashed curve D in Figure 8):

$$k = 4.64 \cdot \left(\frac{a}{b}\right)^{-1.106}$$

$$F_e = \frac{k \cdot \pi^2 \cdot E}{12\left(1 - \nu^2\right) \cdot \left(\frac{b}{t}\right)^2}$$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2 \cdot \sqrt{2}}\right)$$

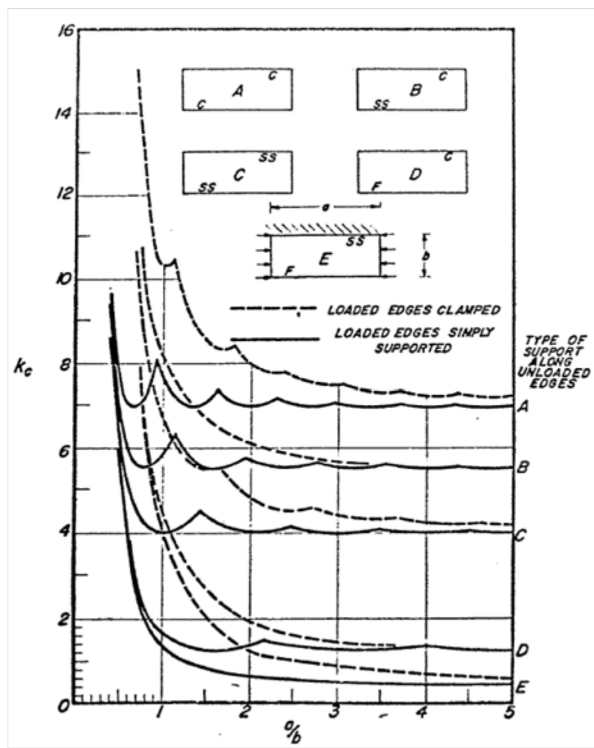


Figure 8: Elastic Buckling Coefficients [2]

Compare calculated principle stress to critical stress.

$$\sigma_{Princ} = \frac{\sigma_v}{2} + \sqrt{\left(\frac{\sigma_v}{2}\right)^2 + \tau_v^2} \quad \leq \quad F_{cr}$$

G3.2.2c BCC Buckling Check Cont.:

Since buckling of the short and long spans are not a concern for the Basic Corner Check, no reduction in calculated capacity is required and capacity calculated using von Mises stress applies.

$$C_{BCC} = 852\,kip$$

$$ORF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{852kip - 1.3 \cdot \left|\frac{1}{2} \cdot -862kip\right|}{1.3 \cdot \left|-215kip\right|} = 2.08$$

$$IRF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{852kip - 1.3 \cdot \left|\frac{1}{2} \cdot -862kip\right|}{2.17 \cdot \left|-215kip\right|} = 1.25$$

BCC Resultant Capacity (per plate)

Total member capacity

$$2 \cdot 852kip = 1703kip$$

The rating factors based on the Basic Corner Check are greater than those based on the Horizontal Shear Check, so no further increases are possible from performing a Refined Corner Check.

[2] George Gerard and Herbert Becker. *Handbook of Structural Stability*, Part I - Buckling of Flat Plates, Tech. Note 3871, National Advisory Committee for Aeronautics, Washington, D.C., July 1957.

Load Factor Rating (LFR) Method

G1.2.5 Evaluation Summary:

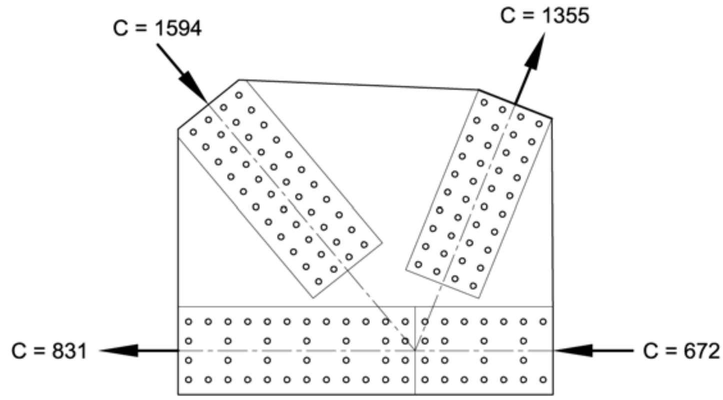


Figure 17: Concurrent Member Capacities Based on Refined Analysis (for Gusset Plate Pair)

Limit State	Gusset Plate Pair	
	Operating Rating	Inventory Rating
Fasteners	2.62	1.57
Vertical Shear	3.60	2.16
~~Horizontal Shear~~[1]	~~1.69~~	~~1.01~~
~~Partial Shear Yield~~[2]	~~1.02~~	~~0.61~~
~~Whitmore Compression~~[2]	~~2.48~~	~~1.48~~
Tension	4.06	2.43
Block Shear	3.99	2.39
Chord Splice	45.9	27.5
Horizontal Shear (Ω Calc.)	1.69	1.01
Basic Corner Check	2.08	1.25

Controls

[1] Superceded by Horizontal Shear with Ω calculated.
[2] Superceded by Basic Corner Check.

By refining the analysis calculations using the approach presented above, a 66% increase in the Operating Rating can be achieved.

Example 4 - Noncompact Gusset Plate with Long Vertical Buckling Length

Load Factor Rating (LFR) Method

Example 4 is a five member gusset plate with a long buckling length for the compression diagonal. It is not a compact gusset plate and no members are chamfered. The calculations apply to one of the two gusset plates.

G4.1 Gusset Plate Material, Geometric, and Loading Properties:

Material Properties

$F_y = 53\text{ksi}$

$F_u = 80\text{ksi}$

$E = 29000\text{ksi}$

$\nu = 0.3$

Properties Thickness

$t = \frac{3}{8}\text{in}$

Member Angles

$\theta_{M2} = 63.43\text{deg}$

$\theta_{M3} = 90\text{deg}$

$\theta_{M4} = 45\text{deg}$

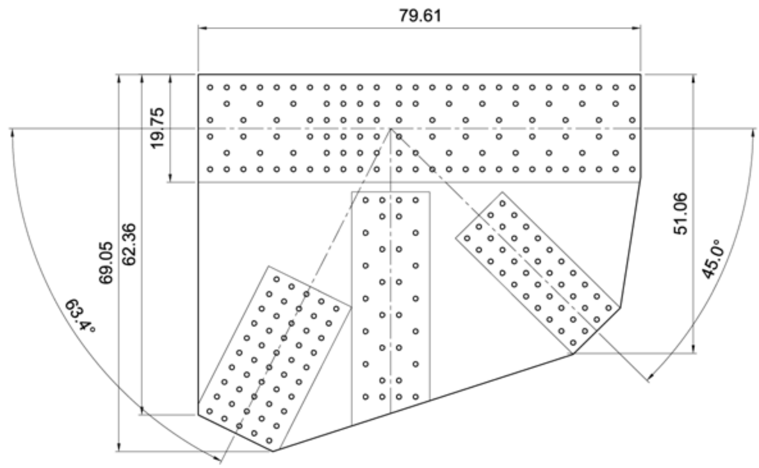


Figure 1: Basic Geometry of Gusset Plate

Unfactored Member Forces Per Gusset Plate Pair

$LL_{M1} = 346\,\text{kip}$

$DL_{M1} = 808\,\text{kip}$

$LL_{M2} = -173\,\text{kip}$

$DL_{M2} = -404\,\text{kip}$

$LL_{M3} = 58\,\text{kip}$

$DL_{M3} = 135\,\text{kip}$

$LL_{M4} = 138\,\text{kip}$

$DL_{M4} = 323\,\text{kip}$

$LL_{M5} = 173\,\text{kip}$

$DL_{M5} = 404\,\text{kip}$

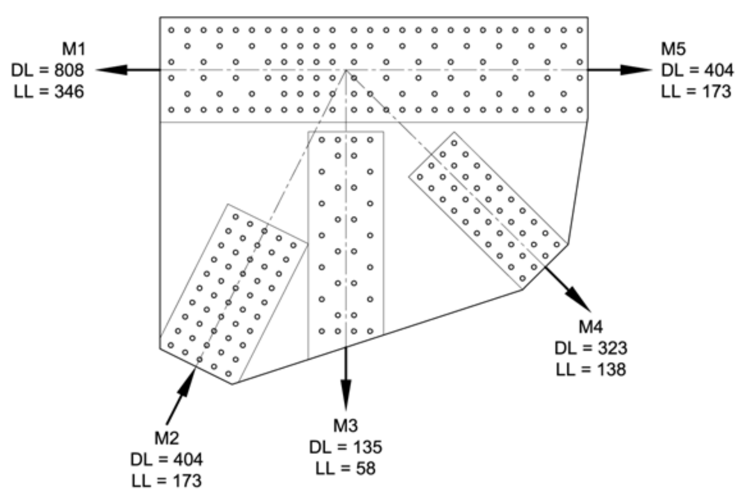


Figure 2: Concurrent Member Forces Transferred to Two Gusset Plates

Member forces based on NCHRP Project 12-84 loads with an assumed Dead Load to Live Load ratio of 70/30.

G4.1 Gusset Plate Material, Geometric, and Loading Properties, Cont.:

Factored Forces Acting on Gusset Plate Pair

$InvForce_{M1} = \gamma_{InvLL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 2.17 \cdot 346kip + 1.3 \cdot 808kip$ $\quad InvForce_{M1} = 1801\,kip$

$OpForce_{M1} = \gamma_{LL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 1.3 \cdot 346kip + 1.3 \cdot 808kip$ $\quad OpForce_{M1} = 1500\,kip$

$InvForce_{M2} = -901\,kip$

$OpForce_{M2} = -750\,kip$

$InvForce_{M3} = 300\,kip$

$OpForce_{M3} = 250\,kip$

$InvForce_{M4} = 720\,kip$

$OpForce_{M4} = 600\,kip$

$InvForce_{M5} = 901\,kip$

$OpForce_{M5} = 750\,kip$

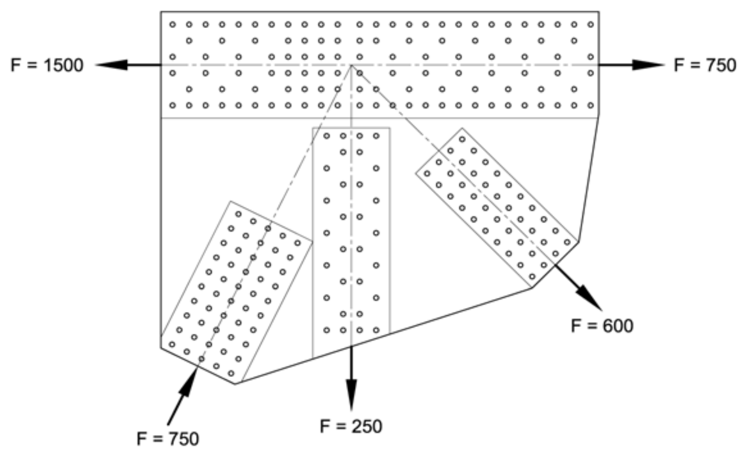


Figure 3: Concurrent Member Operating Forces Transferred to Two Gusset Plates

G4.2 Evaluation Approach:

In accordance with the *2014 Interim Revisions to the Manual for Bridge Evaluation, Second Edition,* the following gusset plate limit state checks were done:
(a) Fastener strength (L6B.2.6.1)
(b) Vertical shear resistance (L6B.2.6.3)
(c) Horizontal shear resistance (L6B.2.6.3)
(d) Partial shear yield resistance (L6B.2.6.3)
(e) Compressive (Whitmore) resistance (L6B.2.6.4)
(f) Tension strength (L6B.2.6.5)
(g) Bock shear resistance (L6B.2.6.5)
(h) Chord splice capacity (L6B.2.6.6)

Load Factor Rating Summary for Example 4

Limit State	Gusset Plate Pair		
	Operating Rating	Inventory Rating	
Fasteners	2.53	1.52	7/8 in. diam A325 threads excluded fasteners
Vertical Shear	3.97	2.38	
Horizontal Shear	4.72	2.82	
Partial Shear Yield	2.32	1.39	
Whitmore Compression	0.56	0.33	**Controls**
Tension	6.35	3.80	
Block Shear	2.73	1.64	
Chord Splice	3.50	2.09	

When the Partial Shear Plane Yield and/or Whitmore Compression capacity checks control and indicate a less than acceptable rating, more rigorous evaluation should be performed.

The following additional rating checks are performed in Example 4:
(1) Horizontal shear capacity - Ω calculated: Supercedes Horizontal Shear with Ω = 0.88.
(2) Basic Corner Check capacity (BCC): Replaces Partial Shear Plane Yield and Whitmore Compression capacity che

G4.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω):

Global shear check along horizontal plane parallel with bottom chord. Shear force calculated using horizontal component of diagonal member forces. Gross section selected at bottom fasteners of diagonal and vertical members to achieve maximum eccentricity. Net section calculated through bottom chord fastener holes. Ω calculated using Drucker formula.

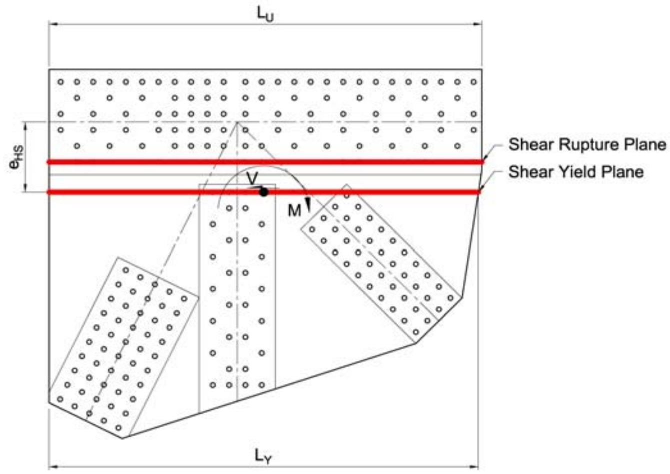


Figure 4: Horizontal Shear Between Web and Chord Members

$L_U = 79.6\,\text{in}$

$L_Y = 79.0\,\text{in}$

$e_{HS} = 13.2\,\text{in}$

$M = V \cdot e_{HS}$

$$A_g = t \cdot L_Y = \frac{3}{8}\text{in} \cdot 79.0\text{in} = 29.6\text{in}^2$$

$d_h = 1\,\text{in}$

$n_{hole} = 26$

$$A_n = t \cdot \left(L - n_{hole} \cdot d_h\right) = \frac{3}{8}\text{in} \cdot [79.6\text{in} - (26) \cdot 1.0\text{in}] = 20.1\text{in}^2$$

Calculate Ω using Drucker formula instead of using AASHTO-specified Ω=0.88

$$V = V_p \cdot \left[1 - \left(\frac{M}{M_p}\right)\right]^{0.25} \qquad V = \Omega \cdot V_p$$

Drucker Formula [1]

$$V_P = (0.58) \cdot F_y \cdot A_g = (0.58) \cdot 53\text{ksi} \cdot 29.6\text{in}^2 = 911\text{kip}$$

$$M_P = \frac{L^2 \cdot t}{4} \cdot F_y = \frac{(79.0\text{in})^2 \cdot \frac{3}{8}\text{in}}{4} \cdot 53\text{ksi} = 31000\text{kip} \cdot \text{in}$$

Substitute $V = \Omega * V_p$ into Drucker formula and rearrange to solve for Ω using plastic shear and moment capacities

$$\Omega \cdot V_p = V_p \cdot \left(1 - \frac{\Omega \cdot V_p \cdot e_{HS}}{M_p}\right)^{0.25}$$

$$\Omega = \left(1 - \frac{\Omega V_P \cdot e_{HS}}{M_P}\right)^{0.25} = \left(1 - \frac{\Omega \cdot 911\text{kip} \cdot 13.2\text{in}}{31000\text{kip} \cdot \text{in}}\right)^{0.25} = 0.90$$

Requires iterative process since V is proportional to Ω. Can substitute AASHTO specified value of Ω = 0.88 on right side of equation as a first estimate of Ω. Result shown is the calculated value of Ω after performing necessary iterations.

G4.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω) Cont.:

$$\phi_{vy} = 1.0$$

$$\phi_{vu} = 0.85$$

$$C_Y = \varphi_{yy} \cdot (0.58) \cdot F_y \cdot A_g \cdot \Omega = 1.00(0.58) \cdot 53\text{ksi} \cdot 29.6\text{in}^2 \cdot (0.90) = 819\text{kip}$$

$$C_U = \varphi_{yu} \cdot (0.58) \cdot F_u \cdot A_n = 0.85(0.58) \cdot 80\text{ksi} \cdot 20.1\text{in}^2 = 793\text{kip}$$

$$C_{HS} = \min\left(C_Y, C_U\right) = \min(819\text{kip}, 793\text{kip}) = 793\text{kip}$$

Horizontal Shear Capacity (per plate)

Note that the horizontal shear capacity is controlled by shear rupture and is not dependent on Ω being 0.88 or calculated.

Determine capacity of member M2 based on Horizontal Shear

$$C_{HS.M2} = C_{HS} \cdot \left|\frac{OpForce_{M2}}{OpForce_{M1} - OpForce_{M5}}\right| = 793\text{kip} \cdot \left|\frac{-750\text{kip}}{1500\text{kip} - (750\text{kip})}\right| = 793\text{kip}$$

Total member capacity
$$2 \cdot 793\text{kip} = 1585\text{kip}$$

$$ORF_{HS} = \frac{C_{HS.M2} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{793\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -404\text{kip}\right|}{1.3 \cdot \left|\frac{1}{2} \cdot -173\text{kip}\right|} = 4.71$$

$$IRF_{HS} = \frac{C_{HS.M2} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{793\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -404\text{kip}\right|}{2.17 \cdot \left|\frac{1}{2} \cdot -173\text{kip}\right|} = 2.82$$

[1] Drucker, D., *The Effect of Shear on the Plastic Bending of Beams*, American Society of Mechanical Engineers, NAMD Conference, Urbana, IL, June 1956

G4.2.2 Basic Corner Check:

The Basic Corner Check is a first-principles analytical approach utilizing fundamental steel design theory to conservatively calculate gusset plate limit state capacities at critical cross sections. This check is used to evaluate equilibrium and stability of a gusset plate "corner" bounded by the bottom chord and vertical member. The "corner" is typically the smallest section encompassing all fasteners of the diagonal member. The diagonal member force is assumed to be resisted by a combination of shear and normal forces acting on the vertical and horizontal surfaces bounding the "corner". Von Mises stress calculated on the surfaces is limited to the yield strength of the gusset plate. For simplicity and to avoid bending in the members, the resultant of each surface must pass through the work point. The "corner" can be adjusted to accomodate deterioration.

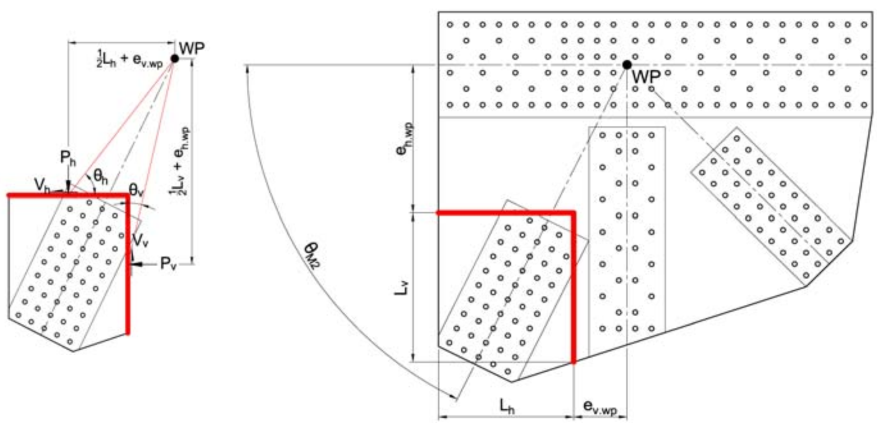


Figure 5: Basic Corner Check for Diagonal M2 Member

Calculate resultant angles from the work point

$L_h = 24.8\,in$ $\qquad e_{h.wp} = 27.63\,in$

$L_v = 27.8\,in$ $\qquad e_{v.wp} = 9.79\,in$

$$\theta_h = atan\left(\frac{e_{h.wp}}{\frac{L_h}{2} + e_{v.wp}}\right) = atan\left(\frac{27.6in}{\frac{24.8in}{2} + 9.79in}\right) = 51.2deg$$

$$\theta_v = atan\left(\frac{e_{v.wp}}{\frac{L_v}{2} + e_{h.wp}}\right) = atan\left(\frac{9.79in}{\frac{27.8in}{2} + 27.63in}\right) = 13.3deg$$

G4.2.2a Horizontal Surface Check:

Since $L_h < L_v$ set von Mises stress on horizontal surface equal to plate yield strength. After stresses on both surfaces are determined, verify assumption that horizontal surface is critical (i.e. reaches von Mises yield before vertical surface).

$$P_h = V_h \cdot \tan(\theta_h)$$

$$\sigma_h = \frac{P_h}{A_h} = \frac{P_h}{L_h \cdot t}$$

$$\tau_h = \frac{V_h}{A_h} = \frac{V_h}{L_h \cdot t}$$

$$\sigma_{vm} = \sqrt{\sigma_h^2 + 3\tau_h^2}$$

Substitute P_h as a function of V_h and set the von Mises stress to yield

$$F_y = 53\text{ksi} = \sigma_{vm} = \sqrt{\sigma_h^2 + 3\tau_h^2} = \sqrt{\left(\frac{P_h}{L_h \cdot t}\right)^2 + 3\cdot\left(\frac{V_h}{L_h \cdot t}\right)^2} = \sqrt{\left(\frac{V_h \cdot \tan(51.2\text{deg})}{24.8\text{in}\cdot\frac{3}{8}\text{in}}\right)^2 + 3\cdot\left(\frac{V_h}{24.8\text{in}\cdot\frac{3}{8}\text{in}}\right)^2}$$

Rearrange terms and solve for V_h

$$V_h = \frac{L_h \cdot F_y \cdot t}{\sqrt{\tan(\theta_h)^2 + 3}} = \frac{24.8\text{in}\cdot 53\text{ksi}\cdot\frac{3}{8}\text{in}}{\sqrt{\tan(51.2\text{deg})^2 + 3}} = 231\text{kip}$$

Solve for P_h

$$P_h = V_h \cdot \tan(\theta_h) = 231\text{kip}\cdot\tan(51.2\text{deg}) = 288\text{kip}$$

Calculate shear and normal stresses on horizontal surface

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{288\text{kip}}{(24.8\text{in})\cdot\left(\frac{3}{8}\text{in}\right)} = 30.9\text{ksi}$$

$$\tau_h = \frac{V_h}{L_h \cdot t} = \frac{231\text{kip}}{(24.8\text{in})\cdot\left(\frac{3}{8}\text{in}\right)} = 24.8\text{ksi}$$

G4.2.2b Vertical Surface Check:

Determine forces and stresses on vertical surface based on horizontal surface forces and stated constraints (i.e. force resultants to pass thru workpoint).

$$P_v = V_v \cdot \tan(\theta_v)$$

$$\theta_v = 13.3 \cdot \deg$$

Constrain final resultant to act along member

$$\theta_{M2} = \operatorname{atan}\left(\frac{V_v + P_h}{P_v + V_h}\right) = \operatorname{atan}\left(\frac{V_v + P_h}{V_v \cdot \tan(\theta_v) + V_h}\right) = \operatorname{atan}\left(\frac{V_v + 288\text{kip}}{V_v \cdot \tan(13.3\deg) + 231\text{kip}}\right)$$

Rearrange terms and solve for V_v. Substitute values obtained from previously solving P_h and V_h.

$$V_v = \frac{P_h - V_h \cdot \tan(\theta_{M2})}{\tan(\theta_{M2}) \cdot \tan(\theta_v) - 1} = \frac{288\text{kip} - 231\text{kip} \cdot \tan(63.4\deg)}{\tan(63.4\deg) \cdot \tan(13.3\deg) - 1} = 331\text{kip}$$

Solve for P_v

$$P_v = V_v \cdot \tan(\theta_v) = 331\text{kip} \cdot \tan(13.3\deg) = 78\text{kip}$$

Calculate shear and normal stresses on vertical surface

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{78\text{kip}}{(27.8\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 7.5\text{ksi}$$

$$\tau_v = \frac{V_v}{L_v \cdot t} = \frac{331\text{kip}}{(27.8\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 31.8\text{ksi}$$

$$\sigma_{vm.v} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{(7.5\text{ksi})^2 + 3 \cdot (31.8\text{ksi})^2} = 55.5\text{ksi} \quad \geq \quad F_y = 53\,\text{ksi}$$

Since the von Mises stress on the vertical surface is greater than the yield strength of the gusset plate, the vertical surface must control. Perform second iteration of calculations while setting the von Mises stress on the vertical surface to the yield stress and then determining the necessary resultants on the horizontal surface to balance the moment about the work point.

G4.2.2b1 Second Iteration - Knowing Vertical Surface Controls:

G4.2.2b2 Determine Forces on Vertical Surface:

Knowing that the vertical surface controls this particular corner check, determine forces acting on vertical surface.

$$P_v = V_v \cdot \tan(\theta_v)$$

Substitute P_v as a function of V_v and set the von Mises stress to yield

$$F_y = 53\text{ksi} = \sigma_{vm} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{\left(\frac{P_v}{L \cdot t}\right)^2 + 3\cdot\left(\frac{V_v}{L \cdot t}\right)^2} = \sqrt{\left(\frac{V_v \cdot \tan(13.3\text{deg})}{27.8\text{in} \cdot \frac{3}{8}\text{in}}\right)^2 + 3\cdot\left(\frac{V_v}{27.8\text{in} \cdot \frac{3}{8}\text{in}}\right)^2}$$

Rearrange terms and solve for V_v

$$V_v = \frac{L_v \cdot F_y \cdot t}{\sqrt{\tan(\theta_v)^2 + 3}} = \frac{27.8\text{in} \cdot 53\text{ksi} \cdot \frac{3}{8}\text{in}}{\sqrt{\tan(13.3\text{deg})^2 + 3}} = 316\text{kip}$$

Solve for P_v

$$P_v = V_v \cdot \tan(\theta_v) = 316\text{kip} \cdot \tan(13.3\text{deg}) = 75\text{kip}$$

Calculate shear and normal stresses on vertical surface (to use when checking buckling strength)

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{75\text{kip}}{(27.8\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 7.2\text{ksi}$$

$$\tau_v = \frac{V_v}{L_v \cdot t} = \frac{316\text{kip}}{(27.8\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 30.3\text{ksi}$$

G4.2.2b3 Determine Forces on Horizontal Surface:

Determine forces and stresses on horizontal surface based on vertical surface forces and stated constraints (i.e. force resultants to pass thru workpoint).

$$P_h = V_h \cdot \tan(\theta_h)$$

Constrain final resultant to act along member and substitute P_h as a function of V_h

$$\theta_{M2} = \operatorname{atan}\left(\frac{V_v + P_h}{P_v + V_h}\right) = \operatorname{atan}\left(\frac{V_v + V_h \cdot \tan(\theta_h)}{P_v + V_h}\right) = \operatorname{atan}\left(\frac{316\text{kip} + V_h \cdot \tan(51.2\text{deg})}{75\text{kip} + V_h}\right)$$

Rearrange terms and solve for V_h. Substitute values obtained from previously solving P_v and V_v.

$$V_h = \frac{V_v - P_v \cdot \tan(\theta_{M2})}{\tan(\theta_{M2}) - \tan(\theta_h)} = \frac{316\text{kip} - 75\text{kip} \cdot \tan(63.4\text{deg})}{\tan(63.4\text{deg}) - \tan(51.2\text{deg})} = 221\text{kip}$$

Solve for P_h

$$P_h = V_h \cdot \tan(\theta_h) = 221\text{kip} \cdot \tan(51.2\text{deg}) = 275\text{kip}$$

Calculate shear and normal stresses on vertical surface

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{275\text{kip}}{(24.8\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 29.5\text{ksi} \qquad \tau_h = \frac{V_h}{L_h \cdot t} = \frac{221\text{kip}}{(24.8\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 23.7\text{ksi}$$

Calculate von Mises stress

$$\sigma_{vm.h} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}} = \sqrt{(29.5\text{ksi})^2 + 3 \cdot (23.7\text{ksi})^2} = 50.6\text{ksi} \qquad \leq \qquad F_y = 53\,\text{ksi}$$

As expected, the von Mises stress on the horizontal surface is less than the yield strength of the gusset plate confirming that the vertical surface controls.

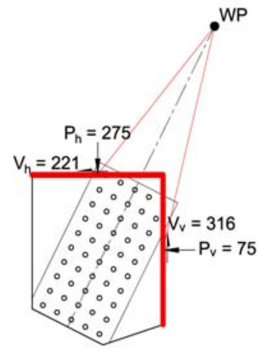


Figure 6: Basic Corner Check Resultants for Diagonal Member M2

$$C_{BCCvM} = \sqrt{(V_h + P_v)^2 + (V_v + P_h)^2} = \sqrt{(221\text{kip} + 75\text{kip})^2 + (316\text{kip} + 275\text{kip})^2} = 660\text{kip}$$

G4.2.2c BCC Buckling Check:

Check plate buckling due to axial forces on Basic Corner Check surfaces (refer to Appendix B). If buckling controls, then von Mises stresses must be adjusted.

Evaluate buckling capacity of plate regions defined by the short and long span between the Basic Corner Check surfaces and adjacent member connections.

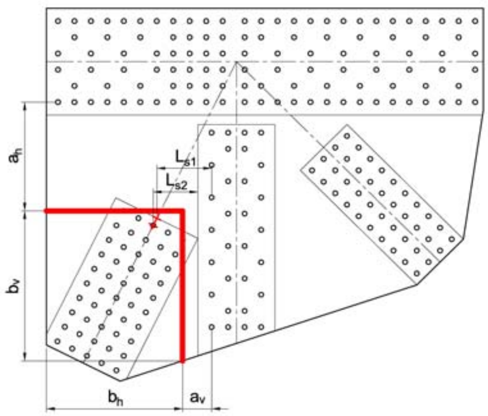


Figure 7: Corner Check Buckling Lengths

G4.2.2c1 Short Span Buckling Check:

For this gusset plate, the short span corresponds to the vertical surface ($a_v < a_h$). a_v and a_h are defined as the distances from the respective Corner Check surface to the parallel line passing through the nearest fastener in an adjacent member.

$$L_s = \frac{L_{s1} + L_{s2}}{2} = \frac{10.0in + 8.2in}{2} = 9.07in$$

$$r = \frac{t}{\sqrt{12}} = \frac{\frac{3}{8}in}{\sqrt{12}} = 0.11in$$

Short span controls sidesway buckling, and rotation at each end is restrained. Therefore, K = 1.0 used.

$$F_e = \frac{\pi^2 \cdot E}{\left(\frac{K \cdot L_s}{r}\right)^2} = \frac{\pi^2 \cdot 29000ksi}{\left(\frac{1.0 \cdot 9.07in}{0.11in}\right)^2} = 40.7ksi$$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2 \cdot \sqrt{2}}\right) = 53ksi \cdot \left(1 - \frac{\sqrt{\frac{53ksi}{40.7ksi}}}{2 \cdot \sqrt{2}}\right) = 31.6ksi$$

G4.2.2c1 Short Span Buckling Check Cont.:

$\sigma = \sigma_v = 7.2\text{ksi}$

$\tau = \tau_v = 31.6\text{ksi}$

$$\sigma_{Principle} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{7.2\text{ksi}}{2} + \sqrt{\left(\frac{7.2\text{ksi}}{2}\right)^2 + (30.3\text{ksi})^2} = 34.1\text{ksi} \qquad \geq \qquad F_{cr} = 31.6\text{ksi}$$

Principle stress is greater than the critical buckling stress; therefore, buckling is a concern and must be addressed.

$$\text{Ratio}_{SG} = \frac{F_{cr}}{\sigma_{Principle}} = \frac{31.6\text{ksi}}{34.1\text{ksi}} = 92.7\%$$

G4.2.2c2 Long Span Buckling Check:
Treat long span as flat rectangular plate with one non-loaded edge fixed and the remaining edges clamped (dashed curve D in Figure 8)

Long Span Length (Figure 8)

$a = 20.1\ \text{in}$

Length of Long Side Surface (Figure 8)

$b = L_h = 24.8\text{in}$

$$\frac{a}{b} = 0.81$$

Because a/b is greater than 0.75 (where k curve is nearly asymptotic), buckling of long span plate may be a concern. Therefore, calculate k as follows (using an approximate best fit function of dashed curve D in Figure 8):

$$k = 4.64\cdot\left(\frac{a}{b}\right)^{-1.106} = 5.85$$

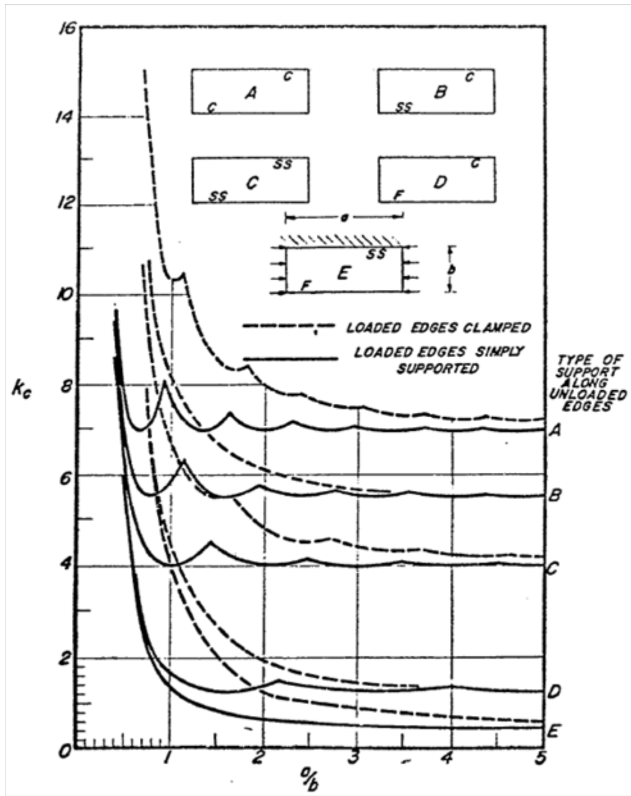


Figure 8: Elastic Buckling Coefficients [2]

$$F_e = \frac{K\cdot\pi^2\cdot E}{12\left(1-\nu^2\right)\cdot\left(\frac{b}{t}\right)^2} = \frac{5.85\cdot\pi^2\cdot 29000\text{ksi}}{12\cdot\left(1-0.3^2\right)\cdot\left(\frac{24.8\text{in}}{\frac{3}{8}\text{in}}\right)^2} = 35.0\text{ksi}$$

Since $F_e = 35.0\text{ksi} > \frac{F_y}{2} = 26.5\text{ksi}$

$$F_{cr} = F_y\cdot\left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2\cdot\sqrt{2}}\right) = 53\text{ksi}\cdot\left(1 - \frac{\sqrt{\frac{53\text{ksi}}{35.0\text{ksi}}}}{2\cdot\sqrt{2}}\right) = 29.9\text{ksi}$$

$\sigma = \sigma_h = 29.5\text{ksi}$

$\tau = \tau_h = 23.7\text{ksi}$

$$\sigma_{\text{Principle}} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{29.5\text{ksi}}{2} + \sqrt{\left(\frac{29.5\text{ksi}}{2}\right)^2 + (23.7\text{ksi})^2} = 42.7\text{ksi} \qquad \geq \qquad F_{cr} = 29.9\text{ksi}$$

G4.2.2c2 Long Span Buckling Check Cont.:

Principle stress is greater than the critical buckling stress; therefore, buckling is a concern and must be addressed.

$$\text{Ratio}_{LG} = \frac{F_{cr}}{\sigma_{Principle}} = \frac{29.9\text{ksi}}{42.7\text{ksi}} = 70.1\%$$

Reduce Capacity Based on Buckling

$$\text{Ratio} = \min\left(\text{Ratio}_{SG}, \text{Ratio}_{LG}\right) = \min(92.5\%, 70.1\%) = 70.1\%$$

$$C_{BCC} = C_{BCCvM} \cdot \text{Ratio} = 660\text{kip} \cdot 70.1\% = 463\text{kip}$$

BCC Resultant Capacity (per plate)

Total member capacity

$$2 \cdot 463\text{kip} = 926\,\text{kip}$$

$$ORF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{463\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -404\text{kip}\right|}{1.3 \cdot \left|-173\text{kip}\right|} = 1.78$$

$$IRF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{463\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -404\text{kip}\right|}{2.17 \cdot \left|-173\text{kip}\right|} = 1.07$$

[2] George Gerard and Herbert Becker. *Handbook of Structural Stability*, Part I - Buckling of Flat Plates, Tech. Note 3871, National Advisory Committee for Aeronautics, Washington, D.C., July 1957.

G4.2.5 Evaluation Summary:

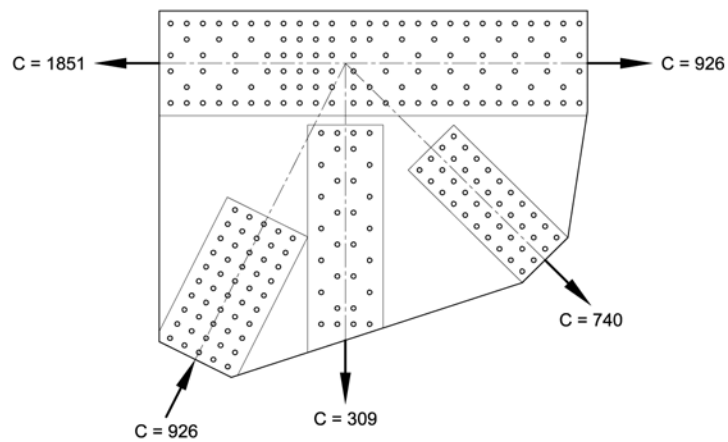


Figure 9: Concurrent Member Capacities Based on Refined Analysis (for Gusset Plate Pair)

Limit State	Gusset Plate Pair	
	Operating Rating	**Inventory Rating**
Fasteners	2.53	1.52
Vertical Shear	3.97	2.38
~~Horizontal Shear~~[1]	~~4.72~~	~~2.82~~
~~Partial Shear Yield~~[2]	~~2.32~~	~~1.39~~
~~Whitmore Compression~~[2]	~~0.56~~	~~0.33~~
Tension	6.35	3.80
Block Shear	2.73	1.64
Chord Splice	3.50	2.09
Horizontal Shear (Ω Calc.)	4.72	2.82
Basic Corner Check	1.78	1.07

Controls

[1] Superceded by Horizontal Shear with Ω calculated.
[2] Superceded by Basic Corner Check.

By refining the analysis calculations using the approach presented above, greater than a 200% increase in the Operating Rating can be achieved. Moreover, the increased capacity achieves a sufficient rating as to not require retrofitting of the gusset plates.

Example 5 - Compact Chamfered Gusset Plate with Short Vertical Buckling Length

Load Factor Rating (LFR) Method

Example 5 is a four member gusset plate with a short buckling length for the compression diagonal. It is a compact gusset plate with the chamfer on the diagonal - near the vertical member. This example assumes that checks related to vertical shear, fastener strength, chord splice, block shear, and tension are of greater capacity than those checked below. It will be worked following the Load Factor Rating Method (LFR) as applied to one of the two gusset plates.

G5.1 Gusset Plate Material, Geometric, and Loading Properties:

Material Properties

$F_y = 53\text{ksi}$

$F_u = 80\text{ksi}$

$E = 29000\text{ksi}$

$\nu = 0.3$

Plate Thickness

$t = \dfrac{3}{8}\text{in}$

Member Angles

$\theta_{M2} = 60.0\text{deg}$

$\theta_{M3} = 91.9\text{deg}$

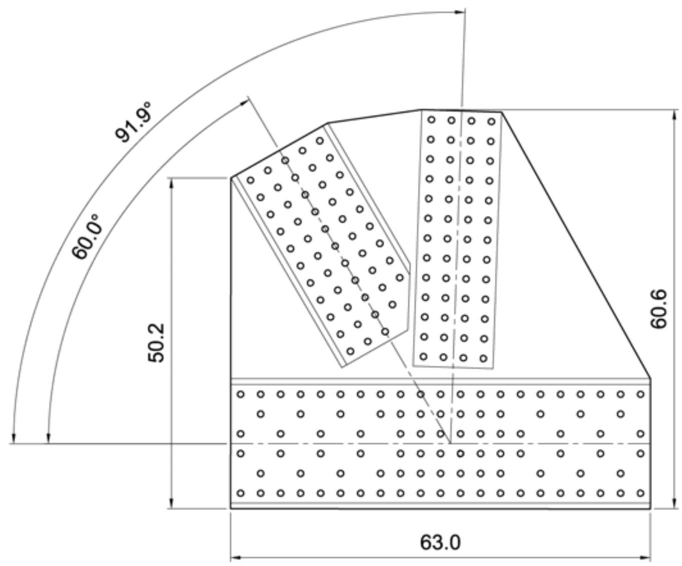


Figure 1: Basic Geometry of Gusset Plate

Unfactored Member Forces Per Gusset Plate Pair

$LL_{M1} = -252\,\text{kip}$

$DL_{M1} = -757\,\text{kip}$

$LL_{M2} = -238\,\text{kip}$

$DL_{M2} = -714\,\text{kip}$

$LL_{M3} = 206\,\text{kip}$

$DL_{M3} = 619\,\text{kip}$

$LL_{M5} = -376\,\text{kip}$

$DL_{M5} = -1129\,\text{kip}$

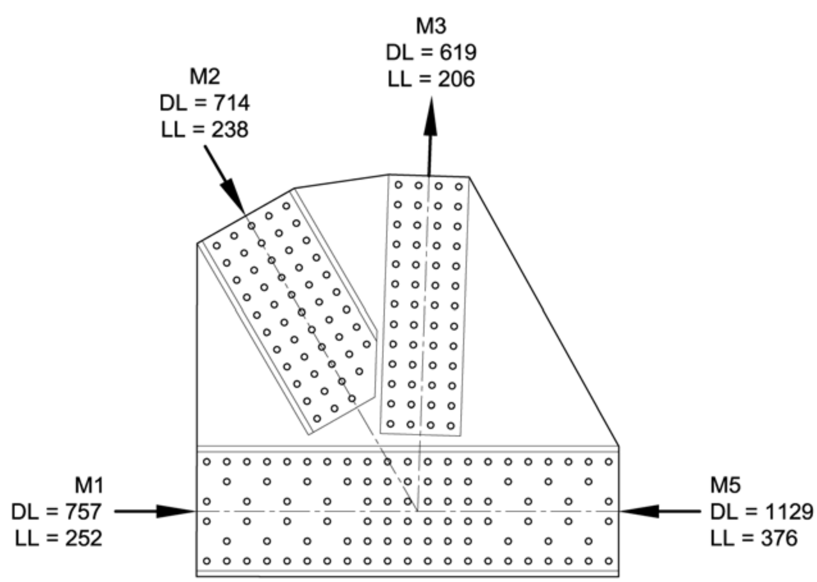


Figure 2: Concurrent Member Forces Transferred to Two Gusset Plates

Member forces based on NCHRP Project 12-84 loads with an assumed Dead Load to Live Load ratio of 75/25.

G5.1 Gusset Plate Material, Geometric, and Loading Properties, Cont.:

Factored Forces Acting on Gusset Plate Pair

$InvForce_{M1} = \gamma_{InvLL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 2.17 \cdot -252kip + 1.3 \cdot -757kip = -1532kip$ $\quad InvForce_{M1} = -1532\,kip$

$OpForce_{M1} = \gamma_{LL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M} = 1.3 \cdot -252kip + 1.3 \cdot -757kip = -1313kip$ $\quad OpForce_{M1} = -1313\,kip$

$InvForce_{M2} = -1445\,kip$

$OpForce_{M2} = -1238\,kip$

$InvForce_{M3} = 1252\,kip$

$OpForce_{M3} = 1073\,kip$

$InvForce_{M5} = -2285\,kip$

$OpForce_{M5} = -1958\,kip$

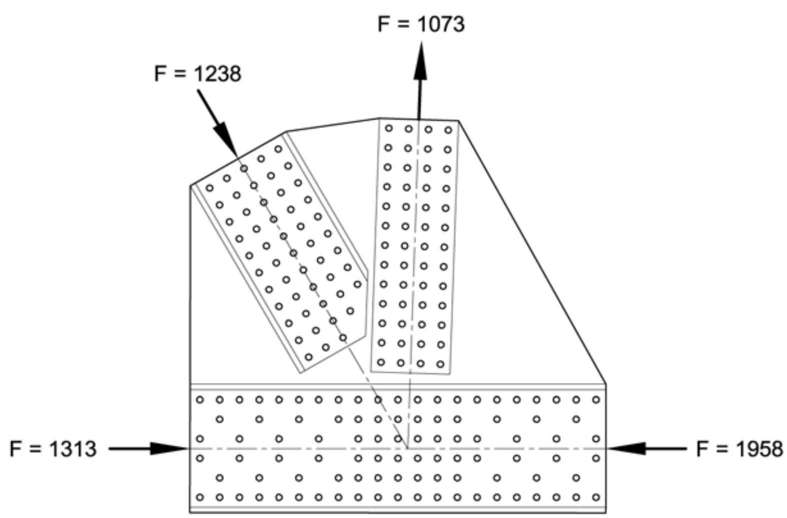


Figure 3: Concurrent Member Operating Forces Transferred to Two Gusset Plates

G5.2 Evaluation Approach:

In accordance with the *2014 Interim Revisions to the Manual for Bridge Evaluation, Second Edition,* the following gusset plate limit state checks were done:
(a) Fastener strength (L6B.2.6.1)
(b) Vertical shear resistance (L6B.2.6.3)
(c) Horizontal shear resistance (L6B.2.6.3)
(d) Partial shear yield resistance (L6B.2.6.3)
(e) Compressive (Whitmore) resistance (L6B.2.6.4)
(f) Tension strength (L6B.2.6.5)
(g) Bock shear resistance (L6B.2.6.5)
(h) Chord splice capacity (L6B.2.6.6)

Load Factor Rating Summary for Example 5

Limit State	Gusset Plate Pair		
	Operating Rating	**Inventory Rating**	
Fasteners	6.41	3.84	7/8 in. diam A325 threads excluded fasteners
Vertical Shear	2.96	1.77	Ω=0.88 with splice plates included
Horizontal Shear	4.71	2.82	
Partial Shear Yield	0.34	0.20	**Controls**
Whitmore Compression	1.49	0.89	
Tension	4.50	2.70	
Block Shear	3.65	2.19	
Chord Splice	7.97	4.77	

When the Partial Shear Plane Yield and/or Whitmore Compression capacity checks control and indicate a less than acceptable rating, more rigorous evaluation should be performed.

The following additional rating checks are performed in Example 5:
(1) Horizontal shear capacity - Ω calculated: Supercedes Horizontal Shear with Ω = 0.88.
(2) Basic Corner Check capacity (BCC): Replaces Partial Shear Plane Yield and Whitmore Compression capacity che
(3) Refined Corner Check capacity (RCC): Supercedes BCC unless BCC indicates acceptable rating.

G5.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω):

Global shear check along horizontal plane parallel with bottom chord. Shear force calculated using horizontal component of diagonal member forces. Gross section selected at bottom fasteners of diagonal and vertical members to achieve maximum eccentricity. Net section calculated through bottom chord fastener holes. Ω calculated using Drucker formula.

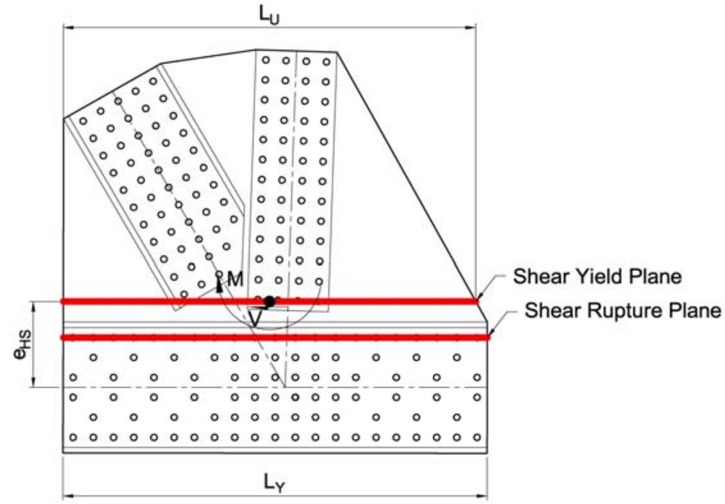


Figure 4: Horizontal Shear between Web and Chord Members

$L_Y = 61.3\,\text{in}$

$L_U = 63.0\,\text{in}$

$e_{HS} = 12.9\,\text{in}$

$M = V \cdot e_{HS}$

$$A_g = t \cdot L_Y = \frac{3}{8}\text{in} \cdot 61.3\text{in} = 23.0\text{in}^2$$

$d_h = 1\,\text{in}$

$n_{hole} = 21$

$$A_n = t \cdot \left(L_U - n_{hole} \cdot d_h\right) = \frac{3}{8}\text{in} \cdot [63.0\text{in} - (21) \cdot 1.0\text{in}] = 15.7\text{in}^2$$

Calculate Ω using Drucker formula instead of using AASHTO-specified Ω=0.88

$$V = V_p \cdot \left[1 - \left(\frac{M}{M_p}\right)\right]^{0.25} \qquad V = \Omega \cdot V_p$$

Drucker Formula [1]

$$V_P = (0.58) \cdot F_y \cdot A_g = (0.58) \cdot 53\text{ksi} \cdot 23.0\text{in}^2 = 706\text{kip}$$

$$M_P = \frac{L_Y^{\,2} \cdot t}{4} \cdot F_y = \frac{(61.3\text{in})^2 \cdot \frac{3}{8}\text{in}}{4} \cdot 53\text{ksi} = 18700\text{in} \cdot \text{kip}$$

Substitute $V = \Omega * V_p$ into Ducker formula and rearrange to solve for Ω using plastic shear and moment capacities

$$\Omega = \left(1 - \frac{V_P \cdot e_{HS} \cdot \Omega}{M_P}\right)^{0.25}$$

$$\Omega = \left(1 - \frac{\Omega \cdot V_P \cdot e_{HS}}{M_P}\right)^{0.25} = \left(1 - \frac{\Omega \cdot 706\text{kip} \cdot 12.9\text{in}}{18700\text{in} \cdot \text{kip}}\right)^{0.25} = 0.87$$

Requires iterative process since V is proportional to Ω. Can substitute AASHTO specified value of Ω = 0.88 on right side of equation as a first estimate of Ω. Result shown is the calculated value of Ω after performing necessary iterations.

G5.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω) Cont.:

$\phi_{vy} = 1.0$

$\phi_{vu} = 0.85$

$C_Y = \varphi_{yy} \cdot (0.58) \cdot F_y \cdot A_g \cdot \Omega = 1.00(0.58) \cdot 53\text{ksi} \cdot 23.0\text{in}^2 \cdot (0.87) = 615\text{kip}$

$C_U = \varphi_{yu} \cdot (0.58) \cdot F_u \cdot A_n = 0.85(0.58) \cdot 80\text{ksi} \cdot 15.7\text{in}^2 = 621\text{kip}$

$C_{HS} = \min(C_Y, C_U) = \min(615\text{kip}, 621\text{kip}) = 615\text{kip}$

Horizontal Shear Capacity (per plate)

Determine capacity of member M2 based on Horizontal Shear

$$C_{HS.M2} = C_{HS} \cdot \left| \frac{OpForce_{M2}}{OpForce_{M1} - OpForce_{M5}} \right| = 615\text{kip} \cdot \left| \frac{-1238\text{kip}}{-1313\text{kip} - (-1958\text{kip})} \right| = 1180\text{kip}$$

Total member capacity
$2 \cdot 1180\text{kip} = 2359\text{kip}$

$$ORF_{HS} = \frac{C_{HS.M2} - \gamma_{DL} \cdot \left| \frac{1}{2} DL_{M2} \right|}{\gamma_{LL} \cdot \left| \frac{1}{2} LL_{M2} \right|} = \frac{1180\text{kip} - 1.3 \cdot \left| \frac{1}{2} \cdot -714\text{kip} \right|}{1.3 \cdot \left| \frac{1}{2} \cdot -238\text{kip} \right|} = 4.63$$

$$IRF_{HS} = \frac{C_{HS.M2} - \gamma_{DL} \cdot \left| \frac{1}{2} DL_{M2} \right|}{\gamma_{InvLL} \cdot \left| \frac{1}{2} LL_{M2} \right|} = \frac{1180\text{kip} - 1.3 \cdot \left| \frac{1}{2} \cdot -714\text{kip} \right|}{2.17 \cdot \left| \frac{1}{2} \cdot -238\text{kip} \right|} = 2.77$$

[1] Drucker, D., *The Effect of Shear on the Plastic Bending of Beams*, American Society of Mechanical Engineers, NAMD Conference, Urbana, IL, June 1956

G5.2.2 Basic Corner Check:

The Basic Corner Check is a first-principles analytical approach utilizing fundamental steel design theory to conservatively calculate gusset plate limit state capacities at critical cross sections. This check is used to evaluate equilibrium and stability of a gusset plate "corner" bounded by horizontal and vertical planes that create the smallest section encompassing all fasteners of the diagonal member. The diagonal member force is assumed to be resisted by a combination of shear and normal forces acting on the vertical and horizontal surfaces bounding the "corner". Von Mises stress calculated on the surfaces is limited to the yield strength of the gusset plate. For simplicity and to avoid bending in the members, the resultant of each surface must pass through the work point. The "corner" can be adjusted in terms of location and plate thickness to accommodate deterioration.

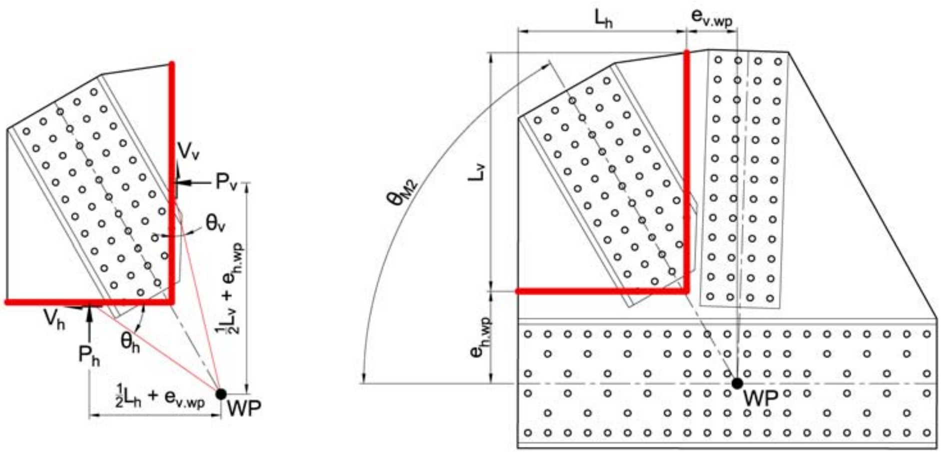


Figure 5: Basic Corner Check for Diagonal Member M2

Calculate resultant angles from the work point

$L_h = 25.3\text{ in}$ $\qquad e_{h.wp} = 14.0\text{ in}$

$L_v = 36.2\text{ in}$ $\qquad e_{v.wp} = 7.62\text{ in}$

$$\theta_h = \text{atan}\left(\frac{e_{h.wp}}{\frac{L_h}{2} + e_{v.wp}}\right) = \text{atan}\left(\frac{14.0\text{in}}{\frac{25.3\text{in}}{2} + 7.62\text{in}}\right) = 34.6\text{deg}$$

$$\theta_v = \text{atan}\left(\frac{e_{v.wp}}{\frac{L_v}{2} + e_{h.wp}}\right) = \text{atan}\left(\frac{7.62\text{in}}{\frac{36.2\text{in}}{2} + 14.0\text{in}}\right) = 13.3\text{deg}$$

G5.2.2a Horizontal Surface Check:

G5.2.2a1 Horizontal Surface Check:

Since $L_h < L_v$ set von Mises stress on horizontal surface equal to plate yield strength. After stresses on both surfaces are determined; verify assumption that horizontal surface is critical (i.e. reaches von Mises yield before vertical surface).

$$P_h = V_h \cdot \tan(\theta_h)$$

$$\sigma_h = \frac{P_h}{A_h} = \frac{P_h}{L_h \cdot t}$$

$$\tau_h = \frac{V_h}{A_h} = \frac{V_h}{L_h \cdot t}$$

$$\sigma_{vm} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}}$$

Substitute P_h as a function of V_h and set the von Mises stress to yield

$$F_y = 53\text{ksi} = \sigma_{vm} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}} = \sqrt{\left(\frac{P_h}{L_h \cdot t}\right)^2 + 3\cdot\left(\frac{V_h}{L_h \cdot t}\right)^2} = \sqrt{\left(\frac{V_h \cdot \tan(34.6\text{deg})}{25.3\text{in}\cdot\frac{3}{8}\text{in}}\right)^2 + 3\cdot\left(\frac{V_h}{25.3\text{in}\cdot\frac{3}{8}\text{in}}\right)^2}$$

Rearrange terms and solve for V_h

$$V_h = \frac{L_h \cdot F_y \cdot t}{\sqrt{\tan(\theta_h)^2 + 3}} = \frac{25.3\text{in}\cdot 53\text{ksi}\cdot\frac{3}{8}\text{in}}{\sqrt{\tan(34.6\text{deg})^2 + 3}} = 270\text{kip}$$

Solve for P_h

$$P_h = V_h \cdot \tan(\theta_h) = 270\text{kip}\cdot\tan(34.6\text{deg}) = 187\text{kip}$$

Calculate shear and normal stresses on horizontal surface

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{187\text{kip}}{(34.6\text{in})\cdot\left(\frac{3}{8}\text{in}\right)} = 19.6\text{ksi}$$

$$\tau_h = \frac{V_h}{L_h \cdot t} = \frac{270\text{kip}}{(25.3\text{in})\cdot\left(\frac{3}{8}\text{in}\right)} = 28.4\text{ksi}$$

G5.2.2b Vertical Surface Check:

Determine forces and stresses on vertical surface based on horizontal surface forces and stated constraints (i.e. force resultants to pass thru workpoint).

$$P_v = V_v \cdot \tan(\theta_v)$$

$$\theta_v = 13.3\cdot\text{deg}$$

Constrain final resultant to act along member

$$\theta_{M2} = \text{atan}\left(\frac{V_v + P_h}{P_v + V_h}\right)$$

G5.2.2b Vertical Surface Check Cont.:

$$\theta_{M2} = \operatorname{atan}\left(\frac{V_v + P_h}{P_v + V_h}\right) = \operatorname{atan}\left(\frac{V_v + P_h}{V_v \cdot \tan(\theta_v) + V_h}\right) = \operatorname{atan}\left(\frac{V_v + 187\text{kip}}{V_v \cdot \tan(13.3\text{deg}) + 270\text{kip}}\right)$$

Rearrange terms and solve for V_v. Substitute values obtained from previously solving P_h and V_h.

$$V_v = \frac{P_h - V_h \cdot \tan(\theta_{M2})}{\tan(\theta_{M2}) \cdot \tan(\theta_v) - 1} = \frac{187\text{kip} - 270\text{kip} \cdot \tan(60.0\text{deg})}{\tan(60.0\text{deg}) \cdot \tan(13.3\text{deg}) - 1} = 478\text{kip}$$

Solve for P_v

$$P_v = V_v \cdot \tan(\theta_v) = 478\text{kip} \cdot \tan(13.3\text{deg}) = 113\text{kip}$$

Calculate shear and normal stresses on vertical surface

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{113\text{kip}}{(36.2\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 8.3\text{ksi} \qquad \tau_v = \frac{V_v}{L_v \cdot t} = \frac{478\text{kip}}{(36.2\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 35.2\text{ksi}$$

$$\sigma_{vm.v} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{(8.3\text{ksi})^2 + 3 \cdot (35.2\text{ksi})^2} = 61.5\text{ksi} \qquad \geq \qquad F_y = 53\,\text{ksi}$$

Since the von Mises stress on the vertical surface is greater than the yield strength of the gusset plate, the vertical surface must control. Perform second iteration of calculations while setting the von Mises stress on the vertical surface to the yield stress and then determining the necessary resultants on the horizontal surface to balance the moment about the work point.

G5.2.2b1 Second Iteration - Knowing Vertical Surface Controls:

G5.2.2b2 Determine Forces on Vertical Surface:

Knowing that the vertical surface controls this particular corner check, determine forces acting on vertical surface.

$$P_v = V_v \cdot \tan(\theta_v)$$

Substitute P_v as a function of V_v and set the von Mises stress to yield

$$F_y = 53\text{ksi} = \sigma_{vm} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{\left(\frac{P_v}{L_v \cdot t}\right)^2 + 3 \cdot \left(\frac{V_v}{L_v \cdot t}\right)^2} = \sqrt{\left(\frac{V_v \cdot \tan(13.3\text{deg})}{36.2\text{in} \cdot \frac{3}{8}\text{in}}\right)^2 + 3 \cdot \left(\frac{V_v}{36.2\text{in} \cdot \frac{3}{8}\text{in}}\right)^2}$$

Rearrange terms and solve for V_v

$$V_v = \frac{L_v \cdot F_y \cdot t}{\sqrt{\tan(\theta_v)^2 + 3}} = \frac{36.2\text{in} \cdot 53\text{ksi} \cdot \frac{3}{8}\text{in}}{\sqrt{\tan(13.3\text{deg})^2 + 3}} = 412\text{kip}$$

Solve for P_v

$$P_v = V_v \cdot \tan(\theta_v) = 412\text{kip} \cdot \tan(13.3\text{deg}) = 98\text{kip}$$

Calculate shear and normal stresses on vertical surface (to use when checking buckling strength)

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{98\text{kip}}{(36.2\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 7.2\text{ksi} \qquad \tau_v = \frac{V_v}{L_v \cdot t} = \frac{412\text{kip}}{(36.2\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 30.3\text{ksi}$$

G5.2.2b3 Determine Forces on Horizontal Surface:

Determine forces and stresses on horizontal surface based on vertical surface forces and stated constraints (i.e. force resultants to pass thru workpoint).

Check the horizontal surface:

$$P_h = V_h \cdot \tan(\theta_h)$$

Constrain final resultant to act along member and substitute P_h as a function of V_h

$$\theta_{M2} = \operatorname{atan}\left(\frac{V_v + P_h}{P_v + V_h}\right) = \operatorname{atan}\left(\frac{V_v + V_h \cdot \tan(\theta_h)}{P_v + V_h}\right) = \operatorname{atan}\left(\frac{412\text{kip} + V_h \cdot \tan(34.6\text{deg})}{98\text{kip} + V_h}\right)$$

Rearrange terms and solve for V_h. Substitute values obtained from previously solving P_v and V_v.

$$V_h = \frac{V_v - P_v \cdot \tan(\theta_{M2})}{\tan(\theta_{M2}) - \tan(\theta_h)} = \frac{412\text{kip} - 98\text{kip} \cdot \tan(60.0\text{deg})}{\tan(60.0\text{deg}) - \tan(34.6\text{deg})} = 233\text{kip}$$

Solve for P_h

$$P_h = V_h \cdot \tan(\theta_h) = 233\text{kip} \cdot \tan(34.6\text{deg}) = 161\text{kip}$$

Calculate shear and normal stresses on vertical surface

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{161\text{kip}}{(25.3\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 16.9\text{ksi} \qquad \tau_h = \frac{V_h}{L_h \cdot t} = \frac{233\text{kip}}{(25.3\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 24.5\text{ksi}$$

Calculate von Mises stress

$$\sigma_{vm.h} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}} = \sqrt{(16.9\text{ksi})^2 + 3 \cdot (24.5\text{ksi})^2} = 45.7\text{ksi} \qquad \leq \qquad F_y = 53\,\text{ksi}$$

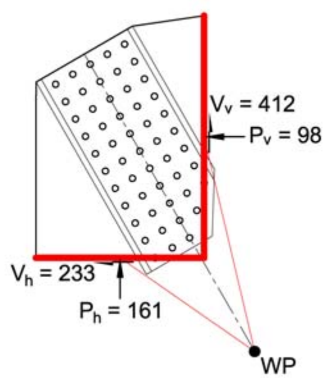


Figure 6: Basic Corner Check Resultants for Diagonal Member M2

$$C_{BCC.vM} = \sqrt{(V_h + P_v)^2 + (V_v + P_h)^2} = \sqrt{(233\text{kip} + 98\text{kip})^2 + (412\text{kip} + 161\text{kip})^2} = 661\text{kip}$$

G5.2.2c BCC Buckling Check:

Check plate buckling due to axial forces on Basic Corner Check surfaces (refer to Appendix B). If buckling controls, then von Mises stresses must be adjusted.

Evaluate buckling capacity of plate regions defined by the short and long span between the Basic Corner Check surfaces and adjacent member connections.

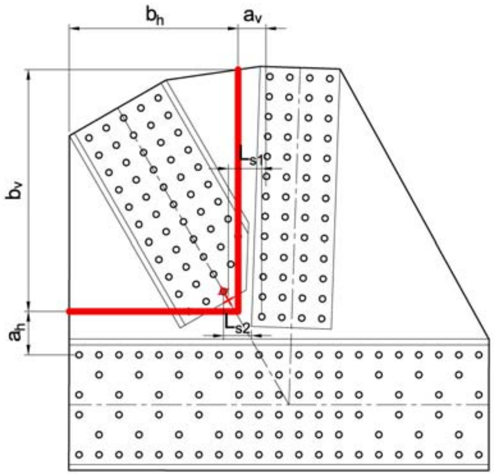


Figure 7: Corner Check Buckling Lengths

G5.2.2c1 Short Span Buckling Check:

For this gusset plate, the short span corresponds to the vertical surface ($a_v < a_h$). a_h and a_v are defined as the distances from the respective Corner Check surface to the parallel line passing through the nearest fastener in an adjacent member.

$$L_s = \frac{L_{s1} + L_{s2}}{2} = \frac{5.0in + 4.2in}{2} = 4.6in$$

$$r = \frac{t}{\sqrt{12}} = \frac{\frac{3}{8}in}{\sqrt{12}} = 0.11in$$

Short span controls sidesway buckling, and rotation at each end is restrained. Therefore, K = 1.0 used.

$$F_e = \frac{\pi^2 \cdot E}{\left(\frac{K \cdot L_s}{r}\right)^2} = \frac{\pi^2 \cdot 29000ksi}{\left(\frac{1.0 \cdot 4.6in}{0.11in}\right)^2} = 158ksi$$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2 \cdot \sqrt{2}}\right) = 53ksi \cdot \left(1 - \frac{\sqrt{\frac{53ksi}{158ksi}}}{2 \cdot \sqrt{2}}\right) = 42.1ksi$$

$$\sigma = \sigma_v = 7.2ksi$$

$$\tau = \tau_v = 30.3ksi$$

$$\sigma_{Principle} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{7.2ksi}{2} + \sqrt{\left(\frac{7.2ksi}{2}\right)^2 + (30.3ksi)^2} = 34.1ksi \qquad \leq \qquad F_{cr} = 42.1ksi$$

Principle stress is less than the critical buckling stress; therefore, buckling is not a concern.

G5.2.2c2 Long Span Buckling Check:

Treat long span as flat rectangular plate with one non-loaded edge fixed and the remaining edges clamped (dashed curve D in Figure 8)

Long Span Length (Figure 8)

$a = 6.5\,\text{in}$

Length of Long Side Surface (Figure 8)

$b = 25.3\,\text{in}$

$$\frac{a}{b} = 0.26$$

Because a/b is less than 0.75 (where k curve is nearly asymptotic), buckling of long span plate is not a concern. Otherwise, calculate k as follows (using an approximate best fit function of dashed curve D in Figure 8):

$$k = 4.64 \cdot \left(\frac{a}{b}\right)^{-1.106}$$

$$F_e = \frac{k \cdot \pi^2 \cdot E}{12\left(1-\nu^2\right)\cdot\left(\frac{b}{t}\right)^2}$$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2\cdot\sqrt{2}}\right)$$

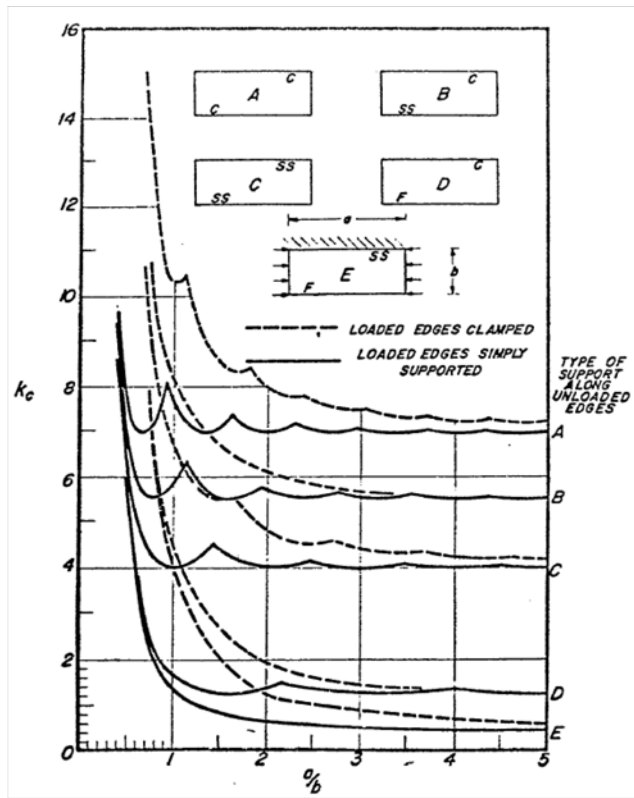


Figure 8: Elastic Buckling Coefficients [2]

Compare calculated principle stress to critical stress.

$$\sigma_{Princ} = \frac{\sigma_v}{2} + \sqrt{\left(\frac{\sigma_v}{2}\right)^2 + \tau_v^2} \quad \le \quad F_{cr}$$

G5.2.2c BCC Buckling Check Cont.:

Since buckling of the short and long spans are not a concern for the Basic Corner Check, no reduction in calculated capacity is required and capacity calculated using von Mises stress applies.

BCC Resultant Capacity (per plate)

$C_{BCC} = 662\ \text{kip}$

Total member capacity

$2 \cdot 662\text{kip} = 1324\text{kip}$

$$ORF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{662\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -714\text{kip}\right|}{1.3 \cdot \left|-238\text{kip}\right|} = 1.28$$

$$IRF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{662\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -714\text{kip}\right|}{2.17 \cdot \left|-238\text{kip}\right|} = 0.77$$

If an increased rating factor is required, perform a Refined Corner Check.

[2] George Gerard and Herbert Becker. *Handbook of Structural Stability*, Part I - Buckling of Flat Plates, Tech. Note 3871, National Advisory Committee for Aeronautics, Washington, D.C., July 1957.

G5.2.3 Refined Corner Check:

The Refined Corner Check removes the constraint that surface resultants pass through the work point as assumed in the Basic Corner Check. In removing this constraint, it is important to check the portion of gusset plate outside of the corner (Stub) and check again for plate buckling based on these resultants.

A typically efficient initial starting point in this iterative check is to force the resultants acting on each surface to be parallel to the member, and then adjust shear and normal forces as necessary.

G5.2.3a Horizontal Surface Check - Parallel Resultants:

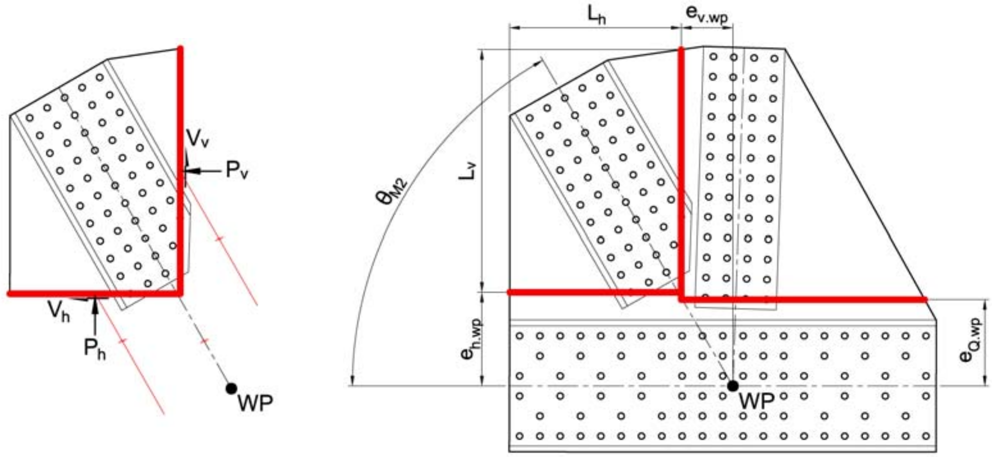


Figure 9: Refined Corner Check for Diagonal Member M2

As with the Basic Corner Check, check to see if the horizontal surface is the controlling surface by setting von Mises stress on horizontal surface equal to plate yield strength. After stresses on both surfaces are determined; verify assumption that horizontal surface is critical (i.e. reaches von Mises yield before vertical surface).

$$V_h = \frac{P_h}{\tan(\theta_{M2})}$$

Constrain von Mises stress on surface equal to the plate yield stress.

$$\sigma_{vm} = \sqrt{\sigma_h^2 + 3\tau_h^2} = F_y$$

$$L_h = 25.3\,in$$

$$\theta_{M2} = 60 \cdot deg$$

Substitute V_h as a function of P_h and set the von Mises stress to yield.

$$F_y = 53ksi = \sigma_{vm} = \sqrt{\sigma_h^2 + 3\tau_h^2} = \sqrt{\left(\frac{P_h}{L_h \cdot t}\right)^2 + 3 \cdot \left(\frac{V_h}{L_h \cdot t}\right)^2} = \sqrt{\left(\frac{P_h}{25.3in \cdot \frac{3}{8}in}\right)^2 + 3 \cdot \left(\frac{\frac{P_h}{\tan(\theta_{M2})}}{25.3in \cdot \frac{3}{8}in}\right)^2}$$

G5.2.3a Horizontal Surface Check Cont.: Parallel Resultants

Rearrange terms and solve for P_h

$$P_h = \frac{F_y \cdot L_h \cdot t \cdot \tan(\theta_{M2})}{\sqrt{\tan(\theta_{M2})^2 + 3}} = \frac{53\text{ksi} \cdot 25.3\text{in} \cdot \frac{3}{8}\text{in} \cdot \tan(60.0\text{deg})}{\sqrt{\tan(60.0\text{deg})^2 + 3}} = 356\text{kip}$$

Solve for V_h

$$V_h = \frac{P_h}{\tan(\theta_{M2})} = \frac{356\text{kip}}{\tan(60.0\text{deg})} = 206\text{kip}$$

Calculate resultants stresses on horizontal surface

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{356\text{kip}}{(25.3\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 37.5\text{ksi} \qquad \tau_h = \frac{V_h}{L_h \cdot t} = \frac{206\text{kip}}{(25.3\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 21.6\text{ksi}$$

G5.2.3b Vertical Surface Check: Parallel Resultants:

Constrain moments about work point to balance (i.e. $\Sigma M_{WP} = 0$)

$$V_v = P_v \cdot \tan(\theta_{M2})$$

$$L_v = 36.2\text{ in}$$

$$e_{v.wp} = 7.6\text{ in}$$

$$e_{h.wp} = 14\text{ in}$$

$$\sum M = 0 = (P_h \cdot e_{p.h} - V_{hi} \cdot e_{v.hi}) - (P_v \cdot e_{p.v} - V_v \cdot e_{v.v})$$

Substitute V_v as a function of P_v, rearrange terms and solve for P_v

$$0 = \left[P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) - V_h \cdot e_{h.wp}\right] - \left[P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right) - P_v \cdot \tan(\theta_{M2}) \cdot e_{v.wp}\right]$$

$$P_v = \frac{\left[P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) - V_h \cdot e_{h.wp}\right]}{\left[\left(\frac{L_v}{2} + e_{h.wp}\right) - \tan(\theta_{M2}) \cdot e_{v.wp}\right]} = \frac{356\text{kip} \cdot \left(\frac{25.3\text{in}}{2} + 7.62\text{in}\right) - 206\text{kip} \cdot (14.0\text{in})}{\left(\frac{36.2\text{in}}{2} + 14.0\text{in}\right) - \tan(45\text{deg}) \cdot 7.62\text{in}} = 229\text{kip}$$

Solve for V_v

$$V_v = P_v \cdot \tan(\theta_{M2}) = 229\text{kip} \cdot \tan(60.0\text{deg}) = 397\text{kip}$$

G5.2.3b Vertical Surface Check Cont.: Parallel Resultants:

Calculate resultants stresses on vertical interface

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{229\text{kip}}{(36.2\text{in})\cdot\left(\frac{3}{8}\text{in}\right)} = 16.9\text{ksi} \qquad \tau_v = \frac{V_v}{L_v \cdot t} = \frac{397\text{kip}}{(36.2\text{in})\cdot\left(\frac{3}{8}\text{in}\right)} = 29.2\text{ksi}$$

Calculate von Mises stress on vertical interface

$$\sigma_{vm.v} = \sqrt{{\sigma_v}^2 + 3{\tau_v}^2} = \sqrt{(16.9\text{ksi})^2 + 3\cdot(29.2\text{ksi})^2} = 53.4\text{ksi} \qquad \geq \qquad F_y = 53\text{ksi}$$

G5.2.3 Refined Corner Check Cont.: Parallel Resultants

Since the von Mises stress on the vertical surface is greater than the yield strength of the gusset plate, the vertical surface actually controls. Calculations could be performed knowing that the vertical surface controls over the horizontal surface, however, because the value is close, the next step performed will be to check the remaining portion of the gusset plate and buckling. If no reduction in capacity is needed due to these checks, then this step will be revisited to determine the distribution of stresses with the vertical surface controlling overall.

$$C_{RCC} = \sqrt{(V_h + P_v)^2 + (V_v + P_h)^2} = \sqrt{(206\text{kip} + 229\text{kip})^2 + (397\text{kip} + 356\text{kip})^2} = 870\text{kip}$$

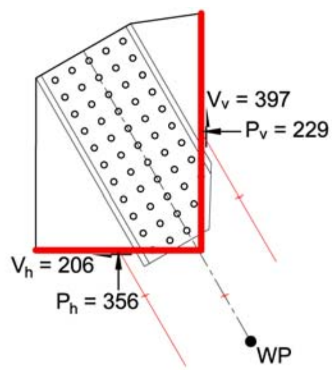


Figure 10: Refined Corner Check Resultants with Parallel Resultants to Member

G5.2.3c Remaining Portion (Stub) Check: Parallel Resultants:

Determine equivalent concurrent forces for vertical member

$$F_{RCC.M3} = OpForce_{M3} \cdot \left| \frac{C_{RCC}}{OpForce_{M2}} \right| = 1073kip \cdot \left| \frac{870kip}{-1238kip} \right| = 754kip$$

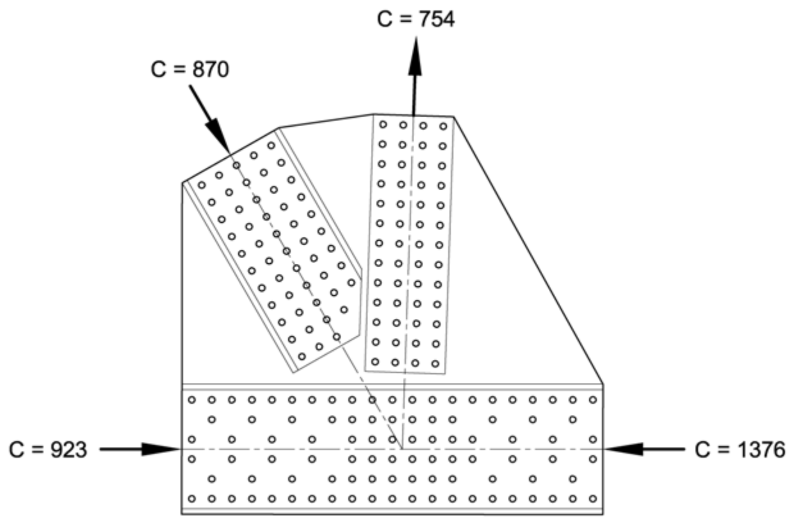


Figure 11: Concurrent Member Capacities (per plate) Based on Refined Corner Check (Subject to Stub Check and Buckling Check)

Check remaining portion of the gusset plate outside of the corner and chord. Select a Section Q that encompasses all force applied by member M3.

$L_Q = 35.9\,in$

$e_{Q.wp} = 12.9\,in$

$\theta_{M3} = 91.9 \cdot deg$

$e_{M3} = 9.9\,in$

$L_v = 36.2\,in$

$e_{h.wp} = 14.0\,in$

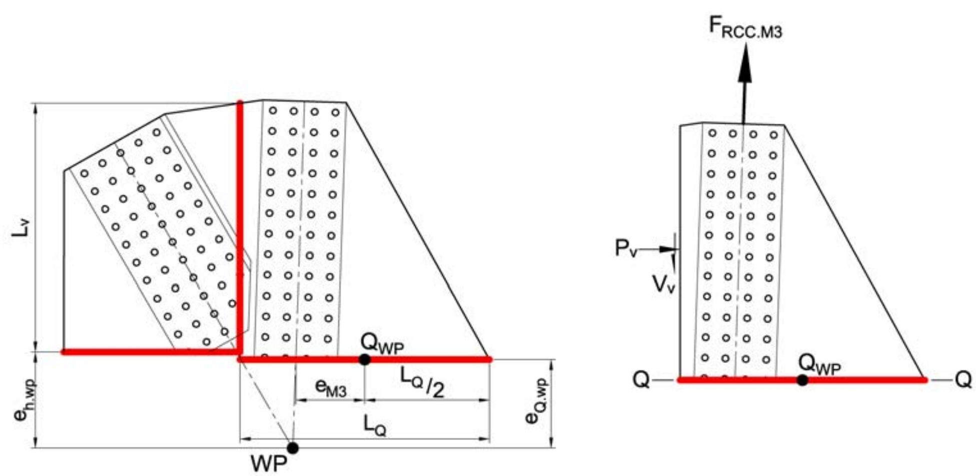


Figure 12: Remaining Gusset Plate Stub

G5.2.3c Remaining Portion (Stub) Check Cont.: Parallel Resultants:

Calculate forces P_Q and V_Q along Section Q

$$P_Q = F_{RCC.M3} \cdot \sin(\theta_{M3}) - V_v = 754kip \cdot \sin(91.9deg) - 397kip = 356kip$$

$$V_Q = -F_{RCC.M3} \cdot \cos(\theta_{M3}) + P_v = -754kip \cdot \cos(91.9deg) + 229kip = 254kip$$

Calculate moment M_Q acting at Q_{WP}

$$M_Q = P_v \cdot \left(\frac{L_v}{2} + e_{h.wp} - e_{Q.wp}\right) - V_v \cdot \frac{L_Q}{2} + F_{RCC.M3} \cdot \sin(\theta_{M3}) \cdot e_{M3}$$

$$M_Q = 229kip \cdot \left(\frac{36.2in}{2} + 14.0in - 12.9in\right) - 397kip \cdot \frac{35.9in}{2} + 754kip \cdot \sin(91.9deg) \cdot 9.9in = 4740kip \cdot in$$

Determine section modulus and calculate bending and normal stresses

$$S = \frac{L_Q^{\,2} \cdot t}{6} = \frac{(35.9in)^2 \cdot \frac{3}{8}in}{6} = 80.7in^3$$

$$\sigma_P = \frac{P_Q}{L_Q \cdot t} = \frac{356kip}{35.9in \cdot \frac{3}{8}in} = 26.4ksi$$

$$\sigma_M = \frac{|M_Q|}{S} = \frac{|4740kip \cdot in|}{80.7in^3} = 58.7ksi$$

One of the restrictions of the Refined Corner Check is to limit the maximum normal stress to the yield stress. With this combination of forces, the stub is overstressed. The input forces will be reduced linearly by this overstress. Note that this also compensates for the vertical surface of the corner being slightly overstressed.

$$Ratio = \frac{F_y}{\sigma_P + \sigma_M} = \frac{53ksi}{26.4ksi + 58.7ksi} = 62.2\%$$

$$\sigma_P = \sigma_P \cdot Ratio = 26.4ksi \cdot 62.2\% = 16.5ksi$$

$$\sigma_M = \sigma_M \cdot Ratio = 58.7ksi \cdot 62.2\% = 36.5ksi$$

$$V_Q = V_Q \cdot Ratio = 229kip \cdot 62.2\% = 158kip$$

Since $\sigma_P + \sigma_M \leq F_y$ and $\sigma_M > \sigma_P$, use σ in von Mises equation based on 0.6*σ_{max} (Refer to Appendix A)

$$\sigma_{0.6} = 0.6 \cdot (\sigma_P + \sigma_M) = 0.6 \cdot (16.5ksi + 36.5ksi) = 31.8ksi$$

$$\Omega = \sqrt{1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2} = \sqrt{1 - \left(\frac{31.8ksi}{53ksi}\right)^2} = 0.80$$

$$\tau_N = \Omega \cdot (0.58) \cdot F_y = 0.80 \cdot (0.58) \cdot 53ksi = 24.6ksi$$

G5.2.3c Remaining Portion (Stub) Check Cont.: Parallel Resultants:

Check shear on Section Q to see if it is less than 24.6 ksi

$$v_Q = \frac{V_Q}{L_Q \cdot t} = \frac{158\text{kip}}{35.9\text{in} \cdot \frac{3}{8}\text{in}} = 11.8\text{ksi} \qquad \le \qquad \tau_N = 24.6\,\text{ksi}$$

Therefore, remaining portion of gusset plate can sustain the demands of the Refined Corner Check

Revise forces acting on corner surfaces

$$P_h = P_h \cdot \text{Ratio} = 356\text{kip} \cdot 62.2\% = 222\text{kip}$$

$$V_h = V_h \cdot \text{Ratio} = 206\text{kip} \cdot 62.2\% = 128\text{kip}$$

$$P_v = P_v \cdot \text{Ratio} = 230\text{kip} \cdot 62.2\% = 143\text{kip}$$

$$V_v = V_v \cdot \text{Ratio} = 398\text{kip} \cdot 62.2\% = 247\text{kip}$$

Note that the von Mises stress on the vertical surface can now be checked to confirm that this reduction has lowered it below the yield stress limit.

$$\sigma_v = \sigma_v \cdot \text{Ratio} = 16.9\text{ksi} \cdot 62.2\% = 10.5\text{ksi}$$

$$\tau_v = \tau_v \cdot \text{Ratio} = 29.2\text{ksi} \cdot 62.2\% = 18.2\text{ksi}$$

$$\sigma_{vm.v} = \sigma_{vm.v} \cdot \text{Ratio} = 53.5\text{ksi} \cdot 62.2\% = 33.2\text{ksi} \qquad \le \qquad F_y = 53\,\text{ksi}$$

This check shows the von Mises stress on the vertical surface is less than the yield strength of the gusset plate due to the reduction necessary for the stub. Essentially, this verifies that the stub does control.

$$C_{RCC} = \sqrt{\left(V_h + P_v\right)^2 + \left(V_v + P_h\right)^2} = \sqrt{(128\text{kip} + 143\text{kip})^2 + (247\text{kip} + 222\text{kip})^2} = 541\text{kip}$$

G5.2.3d RCC Buckling Check: Parallel Resultants:

Check buckling due to axial forces on corner surfaces with Refined Corner Check demands (refer to Appendix B)

G5.2.3d1 Short Span Buckling Check:

For this gusset plate, the short span corresponds to the vertical surface

$F_{cr} = 42.1\text{ksi}$ See Basic Corner Check

$\sigma = \sigma_v = 10.5\text{ksi}$

$\tau = \tau_v = 18.2\text{ksi}$

$$\sigma_{Principle} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{10.5\text{ksi}}{2} + \sqrt{\left(\frac{10.5\text{ksi}}{2}\right)^2 + (18.2\text{ksi})^2} = 24.2\text{ksi} \quad \le \quad F_{cr} = 42.1\text{ksi}$$

The principle stress due to the Refined Corner Check is less than the critical buckling stress; therefore, buckling is not a concern.

G5.2.3d2 Long Span Buckling Check:

Treat as flat rectangular plate with one non loaded edge fixed and the remaining edges clamped

Not a concern as $a/b \le 0.75$ See Basic Corner Check

$C_{RCC} = 541\text{ kip}$

The calculated capacity based on the Refined Corner Check with Parallel Resultants is less than the capacity using the Basic Corner Check. This suggests the RCC force combination is less than optimal and that further refinement of the corner check may be warranted and will likely yield resultants more similar to those of the Basic Corner Check. Because the corner has been controlled by the gusset plate stub, and heavily influenced by the moment demand on the stub, the next steps for analysis refinement will focus on reducing this demand. Note that this combination of forces on the corner results in a lower calculated capacity than following the Basic Corner Check.

G5.2.4 Refined Corner Check: Nonparallel Resultants:

Removing the constraint that the corner surface resultants remain parallel can result in further optimization of the shear and normal forces on the surfaces and an increase in capacity. However, recognize that only a small capacity increase can be gained by further refinement of the analysis before Horizontal Shear controls and that the stub is at capacity.

Allowing the surface resultants to be nonparallel creates multiple equations with multiple unknowns, requiring a complex iterative approach to achieve a solution. In selecting trial values for V and P, recognize that adjustments in shear have a 3x effect on shear stress when considering von Mises stress.

Considering that the stub was controlled by the maximum normal stress at a point, and not the von Mises stress along the plane, consider maximizing stress along one of the surfaces while keeping the peak normal stress on the stub belo\ the yield limit. Knowing that the vertical surface controlled over the horizontal surface for the first iteration, aim to maximize the stress along the vertical surface. To decrease the moment on the stub while increasing calculated capacit an increase to the amount of shear along the vertical surface is necessary.

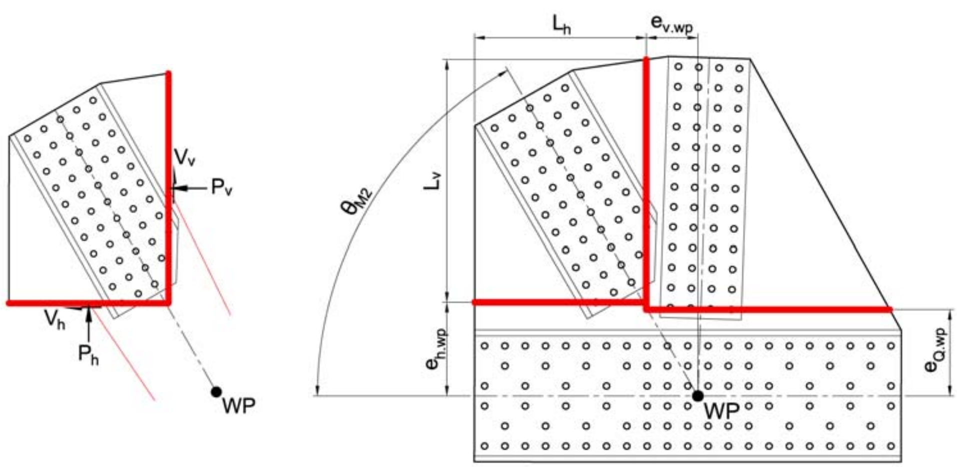


Figure 13: Refined Corner Check for Diagonal Member M2

G5.2.4a Determine Trial Forces and Overall Capacity with All Forces a Function of V_v:

G5.2.4a1 - Horizontal Surface:

Solve the von Mises stress relationship for the axial force on the horizontal surface so that P_v is a function of V_v

$$F_y^{\,2} = \sigma^2 + 3\cdot\tau^2$$

$$F_y^{\,2} = \left(\frac{P_v}{L_v\cdot t}\right)^2 + 3\cdot\left(\frac{V_v}{L_v\cdot t}\right)^2$$

$$P_v = \sqrt{F_y^{\,2}\cdot L_v^{\,2}\cdot t^2 - 3\cdot V_v^{\,2}}$$

G5.2.4a2 Vertical Surface:

Solve for the forces acting on the horizontal surface as a function of the forces acting on the vertical surface

Constrain final resultant to be parallel to member to avoid bending in member

$$\operatorname{atan}\left(\frac{P_h + V_v}{V_h + P_v}\right) = \theta_{M2}$$

Constrain moments about work point to balance (i.e. $\Sigma M_{WP} = 0$)

$$\sum M = 0 = P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) - V_h \cdot e_{h.wp} - P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right) + V_v \cdot e_{v.wp}$$

Solve two equations for P_h and V_h

$$P_h = \tan(\theta_{M2}) \cdot (V_h + P_v) - V_v$$

$$V_h = \frac{P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) + V_v \cdot e_{v.wp} - P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right)}{e_{h.wp}}$$

Substitute for P_h and V_v combine terms and simplify

$$P_h = \tan(\theta_{M2}) \cdot \left[\frac{P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) + V_v \cdot e_{v.wp} - P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right)}{e_{h.wp}} + P_v\right] - V_v$$

$$P_h = \frac{2 \cdot V_v \cdot \left(e_{h.wp} - e_{v.wp} \cdot \tan(\theta_{M2})\right) + P_v \cdot L_v \cdot \tan(\theta_{M2})}{\left(L_h + 2 \cdot e_{v.wp}\right) \cdot \tan(\theta_{M2}) - 2 \cdot e_{h.wp}}$$

G5.2.4a3 Trial Force Substitution:
Choose a value for the shear on the vertical surface (V_v) that gives a calculated capacity above that of Basic Corner Check.

Recall: $C_{BCC} = 662\,kip$ Therefore, select $V_v = 247.26\,kip$

$$P_v = \sqrt{F_y^2 \cdot L_v^2 \cdot t^2 - 3 \cdot V_v^2} = \sqrt{(53.0ksi)^2 \cdot (36.2in)^2 \cdot \left(\frac{3}{8}in\right)^2 - 3 \cdot (400kip)^2} = 197kip$$

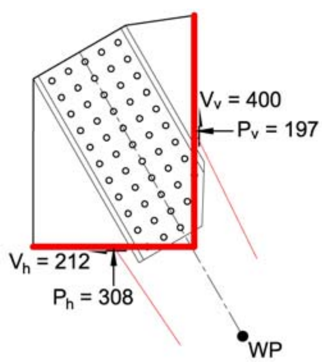
$$P_h = \frac{2 \cdot V_v \cdot \left(e_{h.wp} - e_{v.wp} \cdot \tan\left(\theta_{M2}\right)\right) + P_v \cdot L_v \cdot \tan\left(\theta_{M2}\right)}{\left(L_h + 2 \cdot e_{v.wp}\right) \cdot \tan\left(\theta_{M2}\right) - 2 \cdot e_{h.wp}}$$

$$P_h = \frac{2 \cdot 400kip \cdot (14.0in - 7.6in \cdot \tan(60deg)) + 197kip \cdot 36.2in \cdot \tan(60deg)}{(25.3in + 2 \cdot 7.62in) \cdot \tan(60deg) - 2 \cdot 14.0in} = 308kip$$

$$V_h = \frac{P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) + V_v \cdot e_{v.wp} - P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right)}{e_{h.wp}}$$

$$V_h = \frac{308kip \cdot \left(\frac{25.3in}{2} + 7.62in\right) + 400kip \cdot 7.62in - 197kip \cdot \left(\frac{36.2in}{2} + 14.0in\right)}{14.0in} = 212kip$$

$$C_{RCC} = \sqrt{\left(V_h + P_v\right)^2 + \left(V_v + P_h\right)^2} = \sqrt{(212kip + 197kip)^2 + (400kip + 308kip)^2} = 818kip$$



Total member capacity
$2 \cdot 818kip = 1635kip$

Figure 14: Refined Corner Check Resultants with Resultants Not Parallel to Member

If the stress checks are adequate, this combination of forces would give a capacity greater than that calculated by Basic Corner Check, but still below Horizontal Shear. Proceed knowing that the vertical surface already is at maximum capacity and does not need to be checked.

G5.2.4a4 Horizontal Surface Check: Nonparallel Resultants

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{308\text{kip}}{(25.3\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 32.4\text{ksi} \qquad \tau_h = \frac{V_h}{L_h \cdot t} = \frac{212\text{kip}}{(25.3\text{in}) \cdot \left(\frac{3}{8}\text{in}\right)} = 22.3\text{ksi}$$

$$\sigma_{vm.h} = \sqrt{\sigma_h^{\ 2} + 3\tau_h^{\ 2}} = \sqrt{(32.4\text{ksi})^2 + 3 \cdot (22.3\text{ksi})^2} = 50.4\text{ksi} \qquad \le \ F_y = 53\,\text{ksi}$$

Since the von Mises stress on the horizontal surface is less than the yield strength of the gusset plate, the vertical surface does control over the horizontal surface.

G5.2.4a5 - Remaining Portion (Stub) Check: Nonparallel Resultants

Calculate equivalent concurrent forces for vertical member

$$F_{RCC.M3} = \text{OpForce}_{M3} \cdot \left| \frac{C_{RCC}}{\text{OpForce}_{M2}} \right| = 1073\text{kip} \cdot \left| \frac{818\text{kip}}{-1238\text{kip}} \right| = 709\text{kip}$$

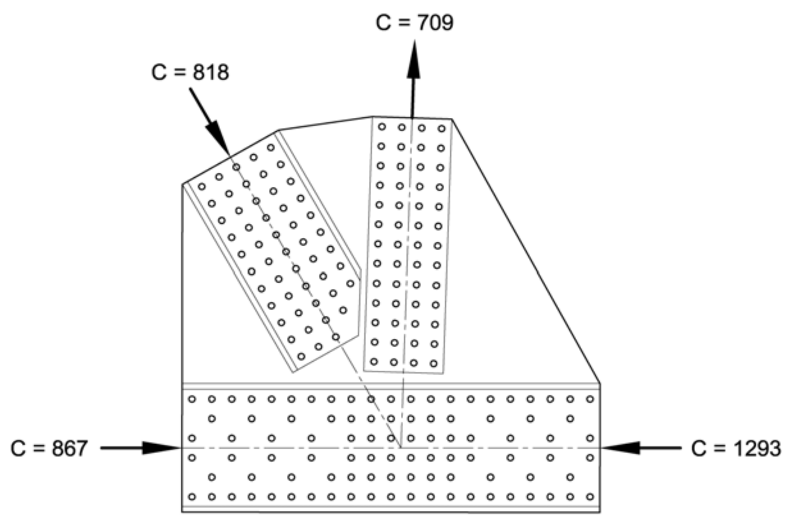


Figure 15: Concurrent Member Capacities (per plate) Based on Refined Corner Check (Subject to Stub Check and Buckling Check)

Check remaining portion of the gusset plate outside of the corner and chord. Select a Section Q that encompasses all force applied by member M3.

$L_Q = 35.9\,\text{in}$

$e_{Q.wp} = 12.9\,\text{in}$

$\theta_{M3} = 91.9 \cdot \text{deg}$

$e_{M3} = 9.9\,\text{in}$

$L_v = 36.2\,\text{in}$

$e_{h.wp} = 14.0\,\text{in}$

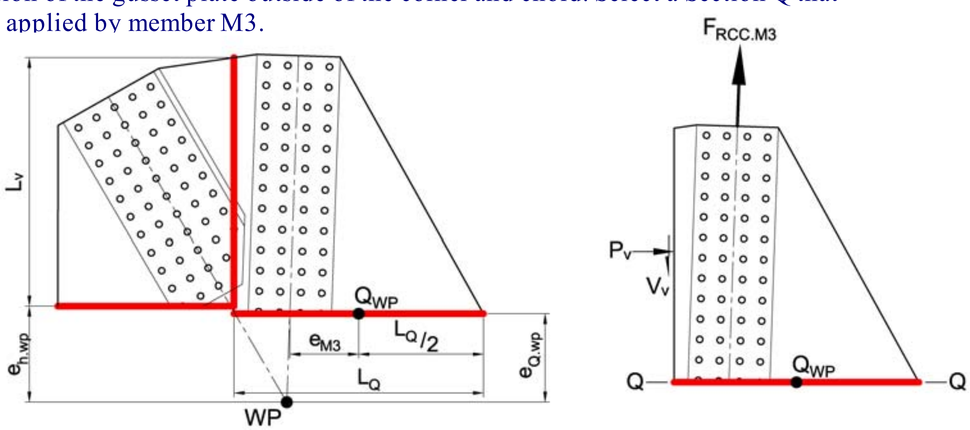


Figure 16: Remaining Gusset Plate Stub

G5.2.4a5 - Remaining Portion (Stub) Check Cont.: Nonparallel Resultants

Calculate forces P_Q and V_Q along Section Q

$$P_Q = F_{RCC.M3} \cdot \sin(\theta_{M3}) - V_v = 709\text{kip} \cdot \sin(91.9\text{deg}) - 400\text{kip} = 308\text{kip}$$

$$V_Q = -F_{RCC.M3} \cdot \cos(\theta_{M3}) + P_v = -708\text{kip} \cdot \cos(91.9\text{deg}) + 197\text{kip} = 220\text{kip}$$

Calculate moment M_Q acting at Q_{WP}

$$M_Q = P_v \cdot \left(\frac{L_v}{2} + e_{h.wp} - e_{Q.wp}\right) - V_v \cdot \frac{L_Q}{2} + F_{RCC.M3} \cdot \sin(\theta_{M3}) \cdot e_{M3}$$

$$M_Q = 197\text{kip} \cdot \left(\frac{36.2\text{in}}{2} + 14.0\text{in} - 12.9\text{in}\right) - 400\text{kip} \cdot \frac{35.9\text{in}}{2} + 709\text{kip} \cdot \sin(91.9\text{deg}) \cdot 9.9\text{in} = 3620\text{kip} \cdot \text{in}$$

$$S = \frac{L_Q^{\ 2} \cdot t}{6} = \frac{(35.9\text{in})^2 \cdot \frac{3}{8}\text{in}}{6} = 80.6\text{in}^3$$

$$\sigma_P = \frac{P_Q}{L_Q \cdot t} = \frac{308\text{kip}}{35.9\text{in} \cdot \frac{3}{8}\text{in}} = 22.9\text{ksi}$$

$$\sigma_M = \frac{|M_Q|}{S} = \frac{|3620\text{kip} \cdot \text{in}|}{80.6\text{in}^3} = 44.9\text{ksi}$$

As before, with the first iteration of the Refined Corner Check, peak normal stress on the stub is greater than the yield stress limit. Input forces could be scaled down as was done previously, but changing the input values directly may be more efficient at calculating a more accurate final capacity. In the next step of this example, a new input value for V_v aiming to decrease further the peak normal stress on the stub will be chosen.

G5.2.4b Second Iteration:

G5.2.4b1 Choose Trial Forces:

Increase the shear on the vertical surface to increase capacity and decrease peak normal stress on the stub.

$$V_v = 405\ \text{kip}$$

$$P_v = \sqrt{F_y^2 \cdot L_v^2 \cdot t^2 - 3 \cdot V_v^2} = \sqrt{(53.0\text{ksi})^2 \cdot (36.2\text{in})^2 \cdot \left(\frac{3}{8}\text{in}\right)^2 - 3 \cdot (405\text{kip})^2} = 163\text{kip}$$

$$P_h = \frac{2 \cdot V_v \cdot \left(e_{h.wp} - e_{v.wp} \cdot \tan\left(\theta_{M2}\right)\right) + P_v \cdot L_v \cdot \tan\left(\theta_{M2}\right)}{\left(L_h + 2 \cdot e_{v.wp}\right) \cdot \tan\left(\theta_{M2}\right) - 2 \cdot e_{h.wp}}$$

$$P_h = \frac{2 \cdot 405\text{kip} \cdot (14.0\text{in} - 7.6\text{in} \cdot \tan(60\text{deg})) + 163\text{kip} \cdot 36.2\text{in} \cdot \tan(60\text{deg})}{(25.3\text{in} + 2 \cdot 7.62\text{in}) \cdot \tan(60\text{deg}) - 2 \cdot 14.0\text{in}} = 259\text{kip}$$

$$V_h = \frac{P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) + V_v \cdot e_{v.wp} - P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right)}{e_{h.wp}}$$

$$V_h = \frac{259\text{kip} \cdot \left(\frac{25.3\text{in}}{2} + 7.62\text{in}\right) + 405\text{kip} \cdot 7.62\text{in} - 163\text{kip} \cdot \left(\frac{36.2\text{in}}{2} + 14.0\text{in}\right)}{14.0\text{in}} = 220\text{kip}$$

$$C_{RCC} = \sqrt{\left(V_h + P_v\right)^2 + \left(V_v + P_h\right)^2} = \sqrt{(220\text{kip} + 163\text{kip})^2 + (405\text{kip} + 259\text{kip})^2} = 766\text{kip}$$

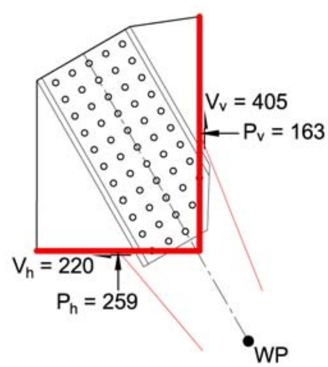


Figure 17: Refined Corner Check Resultants with Resultants Not Parallel to Member

If the stress checks are adequate, this combination of forces would give a capacity greater than that calculated by Basic Corner Check, but still below Horizontal Shear. Proceed knowing that the vertical surface already is at maximum capacity and does not need to be checked.

G5.2.4b2 Horizontal Surface Check: Nonparallel Resultants

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{259\text{kip}}{(25.3\text{in})\cdot\left(\frac{3}{8}\text{in}\right)} = 27.2\text{ksi} \qquad \tau_h = \frac{V_h}{L_h \cdot t} = \frac{220\text{kip}}{(25.3\text{in})\cdot\left(\frac{3}{8}\text{in}\right)} = 23.1\text{ksi}$$

$$\sigma_{vm.h} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}} = \sqrt{(27.2\text{ksi})^2 + 3\cdot(23.1\text{ksi})^2} = 48.4\text{ksi} \qquad \le \; F_y = 53\,\text{ksi}$$

Since the von Mises stress on the horizontal surface is less than the yield strength of the gusset plate, the vertical surface does control over the horizontal surface.

G5.2.4b3 Remaining Portion (Stub) Check Cont.: Nonparallel Resultants

Calculate equivalent concurrent forces for vertical member

$$F_{RCC.M3} = \text{OpForce}_{M3}\cdot\left|\frac{C_{RCC}}{\text{OpForce}_{M2}}\right| = 1073\text{kip}\cdot\left|\frac{766\text{kip}}{-1238\text{kip}}\right| = 664\text{kip}$$

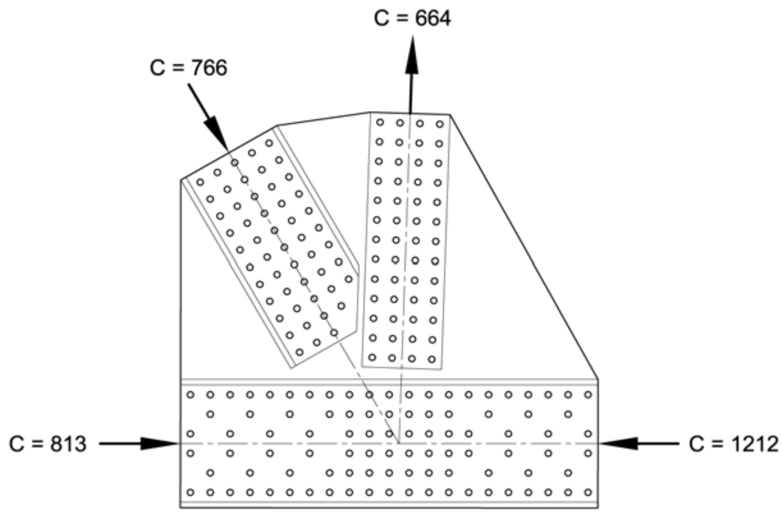


Figure 18: Concurrent Member Capacities (per plate) Based on Refined Corner Check (Subject to Stub Check and Buckling Check)

Check remaining portion of the gusset plate outside of the corner and chord. Select a Section Q that encompasses all force applied by member M3.

$L_Q = 35.9\,\text{in}$

$e_{Q.wp} = 12.9\,\text{in}$

$\theta_{M3} = 91.9\cdot\text{deg}$

$e_{M3} = 9.9\,\text{in}$

$L_v = 36.2\,\text{in}$

$e_{h.wp} = 14.0\,\text{in}$

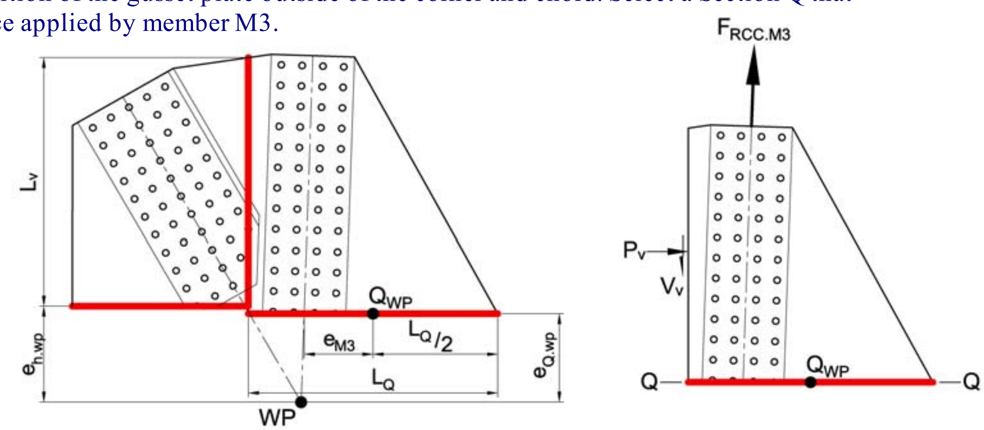


Figure 19: Remaining Gusset Plate Stub

G5.2.4b3 Remaining Portion (Stub) Check Cont.: Nonparallel Resultants

Calculate forces P_Q and V_Q along Section Q

$$P_Q = F_{RCC.M3} \cdot \sin(\theta_{M3}) - V_v = 664kip \cdot \sin(91.9deg) - 405kip = 259kip$$

$$V_Q = -F_{RCC.M3} \cdot \cos(\theta_{M3}) + P_v = -664kip \cdot \cos(91.9deg) + 163kip = 185kip$$

Calculate moment M_Q acting at Q_{WP}

$$M_Q = P_v \cdot \left(\frac{L_v}{2} + e_{h.wp} - e_{Q.wp}\right) - V_v \cdot \frac{L_Q}{2} + F_{RCC.M3} \cdot \sin(\theta_{M3}) \cdot e_{M3}$$

$$M_Q = 163kip \cdot \left(\frac{36.2in}{2} + 14.0in - 12.9in\right) - 405kip \cdot \frac{35.9in}{2} + 664kip \cdot \sin(91.9deg) \cdot 9.9in = 2440kip \cdot in$$

$$S = \frac{L_Q^{\ 2} \cdot t}{6} = \frac{(35.9in)^2 \cdot \frac{3}{8}in}{6} = 80.7in^3$$

$$\sigma_P = \frac{P_Q}{L_Q \cdot t} = \frac{259kip}{35.9in \cdot \frac{3}{8}in} = 19.2ksi$$

$$\sigma_M = \frac{|M_Q|}{S} = \frac{|2440kip \cdot in|}{80.7in^3} = 30.3ksi$$

The peak normal stress on the stub is now less than the yield stress limit. Proceed with checking the von Mises stress on the stub and the plate for buckling.

Since $\sigma_P + \sigma_M < F_y$ and $\sigma_M > \sigma_P$, use σ in von Mises equation based on σ at 0.6*L (Refer to Appendix A)

$$\sigma_{0.6} = 0.6 \cdot (\sigma_P + \sigma_M) = 0.6 \cdot (19.2ksi + 30.3ksi) = 29.7ksi$$

$$\Omega = \sqrt{1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2} = \sqrt{1 - \left(\frac{29.7ksi}{53ksi}\right)^2} = 0.83$$

$$\tau_N = \Omega \cdot (0.58) \cdot F_y = 0.83 \cdot (0.58) \cdot 53ksi = 25.5ksi$$

$$v_Q = \frac{V_Q}{L_Q \cdot t} = \frac{185kip}{35.9in \cdot \frac{3}{8}in} = 13.8ksi \qquad \leq \qquad \tau_N = 25.5\,ksi$$

Therefore, remaining portion of gusset plate is adequate for this combination of forces.

G5.2.4b5 Buckling Check: Nonparallel Resultants

Check buckling due to axial forces on surfaces (refer to Appendix B)

G5.2.4b5a Short Span Buckling Check:

For this gusset plate, the short span corresponds to the vertical surface

$F_{cr} = 42.1\text{ksi}$ See Basic Corner Check

$$\sigma = \sigma_v = \frac{P_v}{L_v \cdot t} = \frac{163\text{ksi}}{36.2\text{in} \cdot \frac{3}{8}\text{in}} = 12.0\text{ksi}$$

$$\tau = \frac{V_v}{L_v \cdot t} = \frac{405\text{kip}}{36.2\text{in} \cdot \frac{3}{8}\text{in}} = 29.8\text{ksi}$$

$$\sigma_{Principle} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{12.0\text{ksi}}{2} + \sqrt{\left(\frac{12.0\text{ksi}}{2}\right)^2 + (29.8\text{ksi})^2} = 36.4\text{ksi} \qquad \le \qquad F_{cr} = 42.1\text{ksi}$$

The principle stress is less than the critical buckling stress; therefore, buckling is not a concern.

G5.2.4d2 Long Span Buckling Check:

Treat as flat rectangular plate with one non loaded edge fixed and the remaining edges clamped

Not a concern as $a/b \le 0.75$ See Basic Corner Check

$C_{RCC} = 766\ \text{kip}$

G5.2.4c Final Iteration:

G5.2.4c1 Choose Trial Forces:

Decrease the shear on the vertical surface while increasing the axial force to increase capacity and maximize the utilization of the stub by increasing peak normal stress on the stub.

$$V_v = 404\,\text{kip}$$

$$P_v = \sqrt{F_y^2 \cdot L_v^2 \cdot t^2 - 3 \cdot V_v^2} = \sqrt{(53.0\text{ksi})^2 \cdot (36.2\text{in})^2 \cdot \left(\frac{3}{8}\text{in}\right)^2 - 3 \cdot (404\text{kip})^2} = 170\text{kip}$$

$$P_h = \frac{2 \cdot V_v \cdot \left(e_{h.wp} - e_{v.wp} \cdot \tan(\theta_{M2})\right) + P_v \cdot L_v \cdot \tan(\theta_{M2})}{\left(L_h + 2 \cdot e_{v.wp}\right) \cdot \tan(\theta_{M2}) - 2 \cdot e_{h.wp}}$$

$$P_h = \frac{2 \cdot 404\text{kip} \cdot (14.0\text{in} - 7.6\text{in} \cdot \tan(60\text{deg})) + 170\text{kip} \cdot 36.2\text{in} \cdot \tan(60\text{deg})}{(25.3\text{in} + 2 \cdot 7.62\text{in}) \cdot \tan(60\text{deg}) - 2 \cdot 14.0\text{in}} = 268\text{kip}$$

$$V_h = \frac{P_h \cdot \left(\frac{L_h}{2} + e_{v.wp}\right) + V_v \cdot e_{v.wp} - P_v \cdot \left(\frac{L_v}{2} + e_{h.wp}\right)}{e_{h.wp}}$$

$$V_h = \frac{268\text{kip} \cdot \left(\frac{25.3\text{in}}{2} + 7.62\text{in}\right) + 404\text{kip} \cdot 7.62\text{in} - 170\text{kip} \cdot \left(\frac{36.2\text{in}}{2} + 14.0\text{in}\right)}{14.0\text{in}} = 218\text{kip}$$

$$C_{RCC} = \sqrt{\left(V_h + P_v\right)^2 + \left(V_v + P_h\right)^2} = \sqrt{(218\text{kip} + 170\text{kip})^2 + (404\text{kip} + 268\text{kip})^2} = 776\text{kip}$$

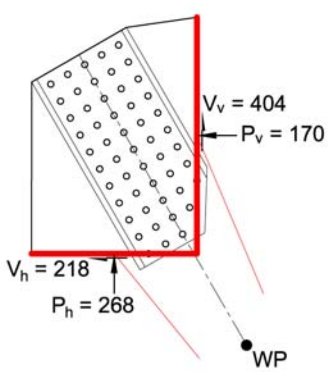


Figure 20: Refined Corner Check Resultants with Resultants Not Parallel to Member

If the stress checks are adequate, this combination of forces would give a capacity much greater than that calculated by Basic Corner Check, but still below Horizontal Shear. Proceed knowing that the vertical surface already is at maximum capacity and does not need to be checked.

G5.2.4c2 Horizontal Surface Check: Nonparallel Results

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{268\text{kip}}{(25.3\text{in})\cdot\left(\frac{3}{8}\text{in}\right)} = 28.2\text{ksi} \qquad \tau_h = \frac{V_h}{L_h \cdot t} = \frac{218\text{kip}}{(25.3\text{in})\cdot\left(\frac{3}{8}\text{in}\right)} = 23.0\text{ksi}$$

$$\sigma_{vm.h} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}} = \sqrt{(28.2\text{ksi})^2 + 3\cdot(22.9\text{ksi})^2} = 48.8\text{ksi} \qquad \leq \; F_y = 53\,\text{ksi}$$

Since the von Mises stress on the horizontal surface is less than the yield strength of the gusset plate, the vertical surface does control over the horizontal surface.

G5.2.4c3 Remaining Portion (Stub) Check: Nonparallel Resultants

Calculate equivalent concurrent forces for vertical member

$$F_{RCC.M3} = OpForce_{M3}\cdot\left|\frac{C_{RCC}}{OpForce_{M2}}\right| = 1073\text{kip}\cdot\left|\frac{776\text{kip}}{-1238\text{kip}}\right| = 673\text{kip}$$

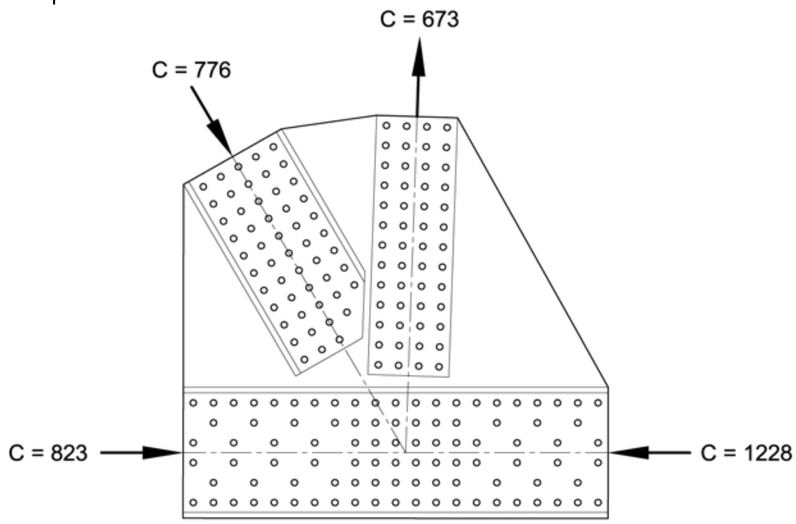


Figure 21: Concurrent Member Capacities (per plate) Based on Refined Corner Check (Subject to Stub Check and Buckling Check)

Check remaining portion of the gusset plate outside of the corner and chord. Select a Section Q that encompasses all force applied by member M3.

$L_Q = 35.9\,\text{in}$

$e_{Q.wp} = 12.9\,\text{in}$

$\theta_{M3} = 91.9\cdot\text{deg}$

$e_{M3} = 9.9\,\text{in}$

$L_v = 36.2\,\text{in}$

$e_{h.wp} = 14.0\,\text{in}$

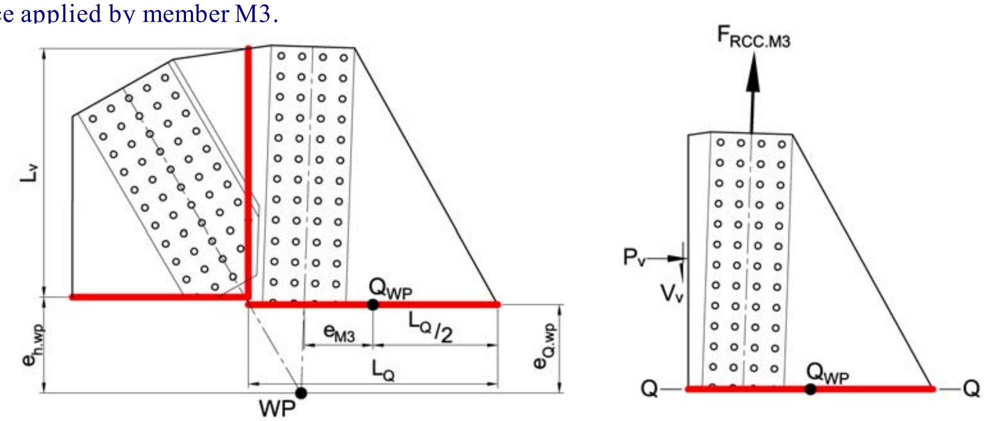


Figure 22: Remaining Gusset Plate Stub

G5.2.4c3 Remaining Portion (Stub) Check: Nonparallel Resultants

Calculate forces P_Q and V_Q along Section Q

$$P_Q = F_{RCC.M3} \cdot \sin(\theta_{M3}) - V_v = 673kip \cdot \sin(91.9deg) - 404kip = 268kip$$

$$V_Q = -F_{RCC.M3} \cdot \cos(\theta_{M3}) + P_v = -673kip \cdot \cos(91.9deg) + 170kip = 192kip$$

Calculate moment M_Q acting at Q_{WP}

$$M_Q = P_v \cdot \left(\frac{L_v}{2} + e_{h.wp} - e_{Q.wp}\right) - V_v \cdot \frac{L_Q}{2} + F_{RCC.M3} \cdot \sin(\theta_{M3}) \cdot e_{M3}$$

$$M_Q = 170kip \cdot \left(\frac{36.2in}{2} + 14.0in - 12.9in\right) - 404kip \cdot \frac{35.9in}{2} + 673kip \cdot \sin(91.9deg) \cdot 9.9in = 2670kip \cdot in$$

$$S = \frac{L_Q{}^2 \cdot t}{6} = \frac{(35.9in)^2 \cdot \frac{3}{8}in}{6} = 80.7in^3$$

$$\sigma_P = \frac{P_Q}{L_Q \cdot t} = \frac{269kip}{35.9in \cdot \frac{3}{8}in} = 19.9ksi$$

$$\sigma_M = \frac{|M_Q|}{S} = \frac{|2670kip \cdot in|}{80.7in^3} = 33.1ksi$$

The peak normal stress on the stub is now equal to the yield stress limit. Proceed with checking the von Mises stress on the stub and the plate for buckling.

Since $\sigma_P + \sigma_M \leq F_y$ and $\sigma_M > \sigma_P$, use σ in von Mises equation based on σ at 0.6*L (Refer to Appendix A)

$$\sigma_{0.6} = 0.6 \cdot (\sigma_P + \sigma_M) = 0.6 \cdot (19.9ksi + 33.0ksi) = 31.8ksi$$

$$\Omega = \sqrt{1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2} = \sqrt{1 - \left(\frac{31.8ksi}{53ksi}\right)^2} = 0.80$$

$$\tau_N = \Omega \cdot (0.58) \cdot F_y = 0.80 \cdot (0.58) \cdot 53ksi = 24.6ksi$$

$$v_Q = \frac{V_Q}{L_Q \cdot t} = \frac{192kip}{35.9in \cdot \frac{3}{8}in} = 14.3ksi \qquad \leq \qquad \tau_N = 24.6\,ksi$$

Therefore, remaining portion of gusset plate is adequate for this combination of forces.

G5.2.4c4 Buckling Check: Nonparallel Resultants

Check buckling due to axial forces on surfaces (refer to Appendix B)

G5.2.4c4a Short Span Buckling Check:

For this gusset plate, the short span corresponds to the vertical surface

$F_{cr} = 42.1\text{ksi}$ See Basic Corner Check

$$\sigma = \sigma_v = \frac{P_v}{L_v \cdot t} = \frac{170\text{ksi}}{36.2\text{in} \cdot \frac{3}{8}\text{in}} = 12.5\text{ksi}$$

$$\tau = \frac{V_v}{L_v \cdot t} = \frac{404\text{kip}}{36.2\text{in} \cdot \frac{3}{8}\text{in}} = 29.7\text{ksi}$$

$$\sigma_{Principle} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{12.5\text{ksi}}{2} + \sqrt{\left(\frac{12.5\text{ksi}}{2}\right)^2 + (29.7\text{ksi})^2} = 36.6\text{ksi} \qquad \le \qquad F_{cr} = 42.1\text{ksi}$$

The principle stress is less than the critical buckling stress; therefore, buckling is not a concern.

G5.2.4c4b Long Span Buckling Check:
Treat as flat rectangular plate with one non loaded edge fixed and the remaining edges clamped

Not a concern as $a/b \le 0.75$ See Basic Corner Check

$C_{RCC} = 776\text{ kip}$

RCC Nonparallel Resultants Capacity (per plate)

Total member capacity
$2 \cdot 776\text{kip} = 1553\text{kip}$

$$ORF_{RCC} = \frac{C_{RCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{776\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -714\text{kip}\right|}{1.3 \cdot |-238\text{kip}|} = 2.02$$

$$IRF_{RCC} = \frac{C_{RCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{776\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -714\text{kip}\right|}{2.17 \cdot |-238\text{kip}|} = 1.21$$

This solution increases ratings to a level that is likely acceptable.

G5.2.5 Evaluation Summary:

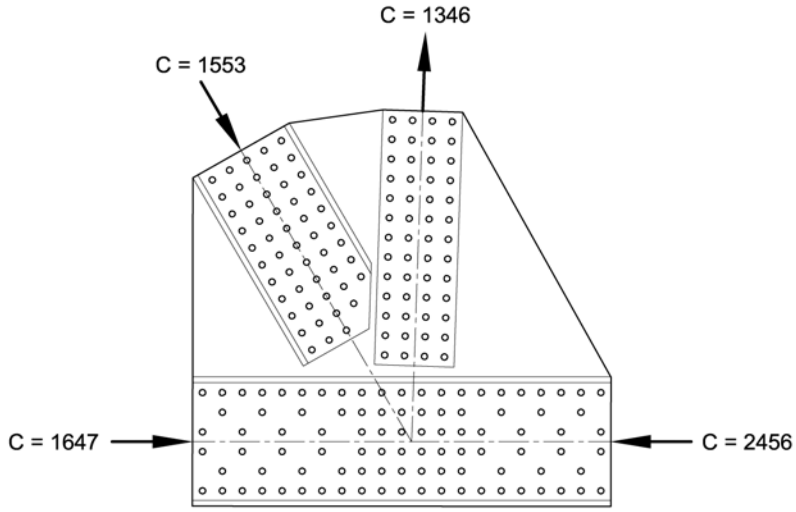


Figure 23: Concurrent Member Capacities Based on Refined Analysis (for Gusset Plate Pair)

Limit State	Gusset Plate Pair	
	Operating Rating	Inventory Rating
Fasteners	6.41	3.84
Vertical Shear	2.96	1.77
~~Horizontal Shear~~[1]	~~4.71~~	~~2.82~~
~~Partial Shear Yield~~[2]	~~0.34~~	~~0.20~~
~~Whitmore Compression~~[2]	~~1.49~~	~~0.89~~
Tension	4.50	2.70
Block Shear	3.65	2.19
Chord Splice	7.97	4.77
Horizontal Shear (Ω Calc.)	4.63	2.77
~~Basic Corner Check~~[3]	~~1.28~~	~~0.77~~
Refined Corner Check	2.02	1.21

Controls

[1] Superceded by Horizontal Shear with Ω calculated.
[2] Superceded by Basic Corner Check (see [3]).
[3] Superceded by final iteration of Refined Corner Check.

By refining the analysis calculations using the approach presented above, the Operating Rating is increased by 500%. Possible repairs to the gusset plate to achive an appropriate load rating are no longer required.

Example 6 - Noncompact Gusset Plate with Medium Vertical Buckling Length and Deterioration

Load Factor Rating (LFR) Method

Example 6 is a four member gusset plate (no vertical) with a medium buckling length between diagonals. It is not a compact gusset plate and no members are chamfered. A band of deterioration exists just above the bottom chord, below the compression diagonal. The calculations apply to one of the two gusset plates.

G6.1 Gusset Plate Material, Geometric, and Loading Properties:

Material Properties

$F_y = 53\text{ksi}$

$F_u = 80\text{ksi}$

$E = 29000\text{ksi}$

$\nu = 0.3$

Plate Thickness

$t = \frac{1}{2}\text{in}$

Member Angles

$\theta_{M2} = 50.47\text{deg}$

$\theta_{M3} = 68.23\text{deg}$

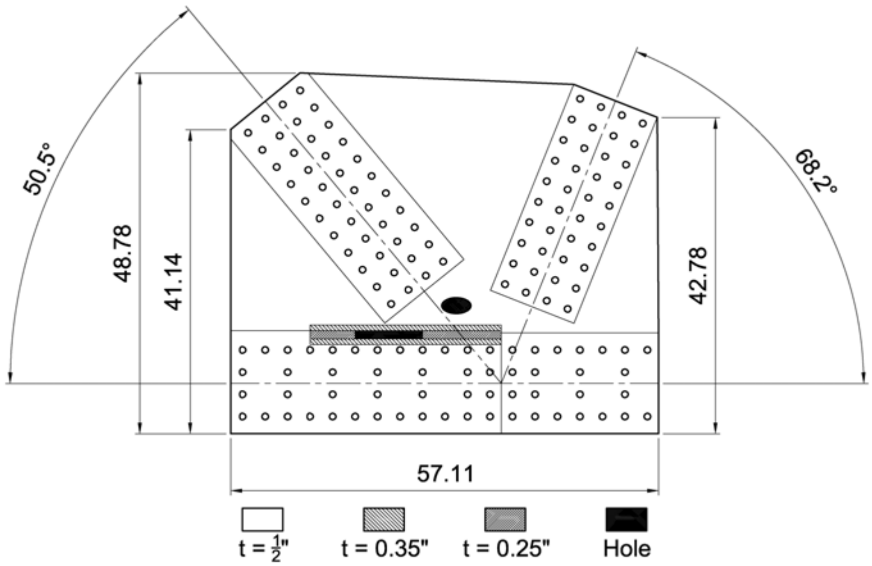


Figure 1: Basic Geometry of Gusset Plate

Unfactored Member Forces Per Gusset Plate Pair

$LL_{M1} = 84\,\text{kip}$

$DL_{M1} = 337\,\text{kip}$

$LL_{M2} = -162\,\text{kip}$

$DL_{M2} = -646\,\text{kip}$

$LL_{M3} = 137\,\text{kip}$

$DL_{M3} = 549\,\text{kip}$

$LL_{M5} = -68\,\text{kip}$

$DL_{M5} = -272\,\text{kip}$

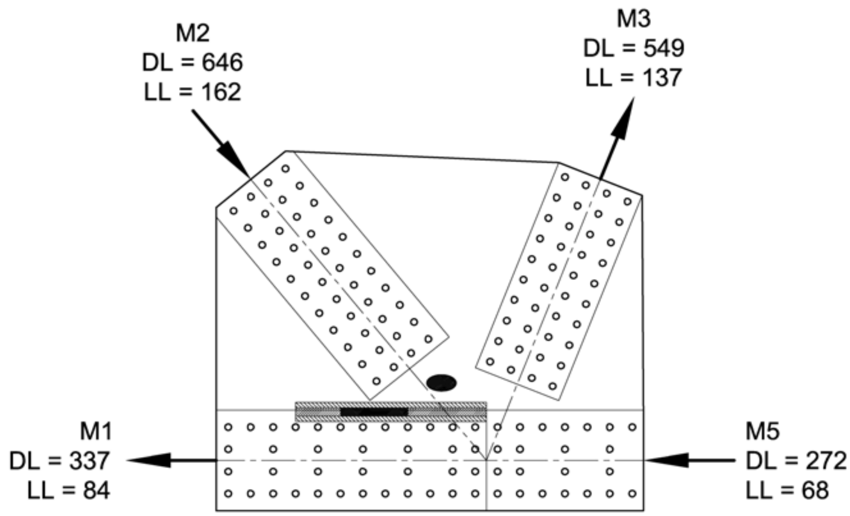


Figure 2: Concurrent Member Forces Transferred to Two Gusset Plates

Member forces based on NCHRP Project 12-84 loads with an assumed Dead Load to Live Load ratio of 80/20.

G6.1 Gusset Plate Material, Geometric, and Loading Properties Cont.:

Factored Forces Acting on Gusset Plate Pair

$InvForce_{M1} = \gamma_{InvLL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 2.17 \cdot 84kip + 1.3 \cdot 337kip$ $\qquad InvForce_{M1} = 621\ kip$

$OpForce_{M1} = \gamma_{LL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 1.3 \cdot 84kip + 1.3 \cdot 337kip$ $\qquad OpForce_{M1} = 548\ kip$

$InvForce_{M2} = -1191\ kip$

$OpForce_{M2} = -1050\ kip$

$InvForce_{M3} = 1012\ kip$

$OpForce_{M3} = 893\ kip$

$InvForce_{M5} = -502\ kip$

$OpForce_{M5} = -443\ kip$

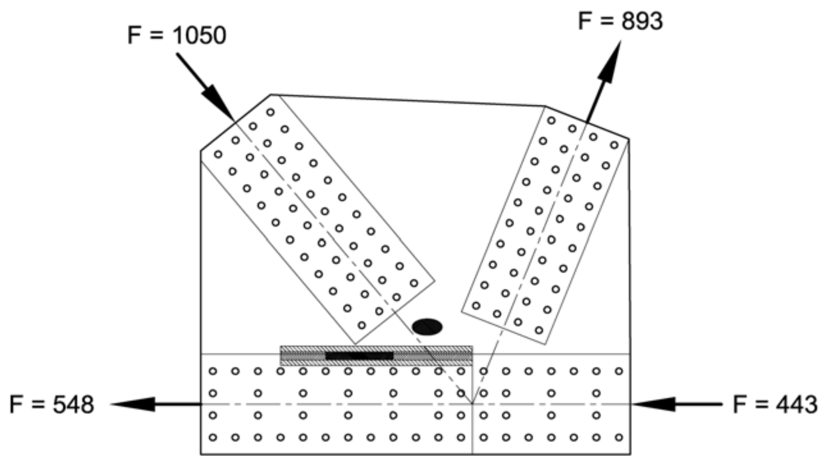


Figure 3: Concurrent Member Operating Forces Transferred to Two Gusset Plates

G6.1 Gusset Plate Material, Geometric, and Loading Properties Cont.:

Deterioration Defined

$L_{AA} = 10.6 \text{ in}$

$L_{BB} = 6 \text{ in}$

$L_{CC} = 9 \text{ in}$

$L_{DD} = 10.5 \text{ in}$

$L_{EE} = 21 \text{ in}$

$t_{AA} = t = \frac{1}{2} \text{in}$

$t_{mid.BB} = 0.25 \text{ in}$

$t_{adj.BB} = 0.35 \text{ in}$

$t_{CC} = 0 \text{in}$

$t_{mid.DD} = 0.25 \text{ in}$

$t_{adj.DD} = 0.35 \text{ in}$

$t_{EE} = t = \frac{1}{2} \text{in}$

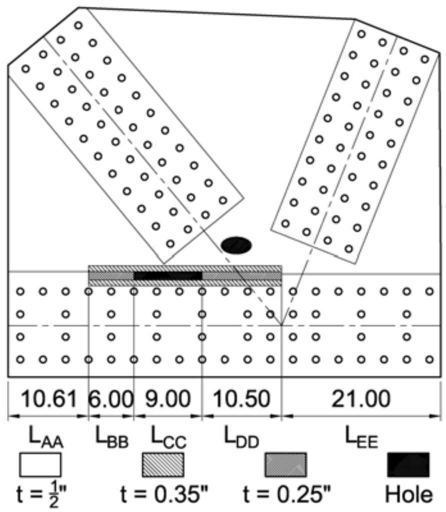


Figure 4: Deterioration Lengths along Horizontal Defined

$L_{WW} = 30.1 \text{ in}$

$L_{XX} = 1.09 \text{ in}$

$L_{YY} = 2.08 \text{ in}$

$L_{ZZ} = 0.75 \text{ in}$

$t_{WW} = t = \frac{1}{2} \text{in}$

$t_{XX} = 0 \text{in}$

$t_{YY} = t = \frac{1}{2} \text{in}$

$t_{ZZ} = 0.35 \text{ in}$

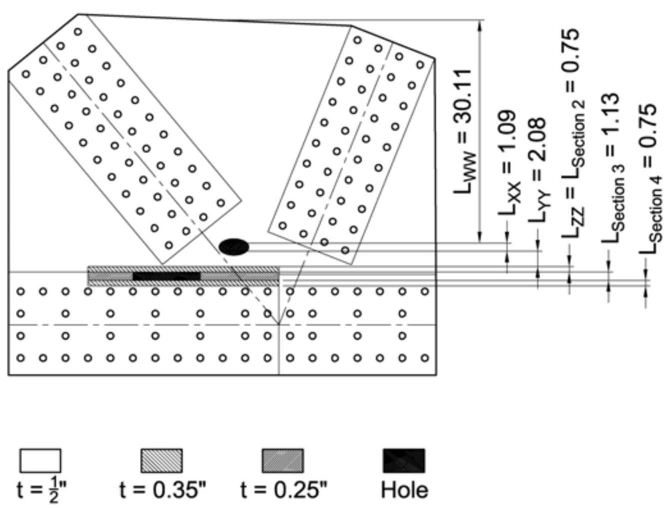


Figure 5: Deterioration Lengths along Vertical Surface of and Short Gap Buckling for Corner Check Defined

G6.2 Evaluation Approach:

In accordance with the *2014 Interim Revisions to the Manual for Bridge Evaluation, Second Edition,* the following gusset plate limit state checks were done:
(a) Fastener strength (L6B.2.6.1)
(b) Vertical shear resistance (L6B.2.6.3)
(c) Horizontal shear resistance (L6B.2.6.3)
(d) Partial shear yield resistance (L6B.2.6.3)
(e) Compressive (Whitmore) resistance (L6B.2.6.4)
(f) Tension strength (L6B.2.6.5)
(g) Bock shear resistance (L6B.2.6.5)
(h) Chord splice capacity (L6B.2.6.6)

Load Factor Rating Summary for Example 6

Limit State	Gusset Plate Pair		
	Operating Rating	**Inventory Rating**	
Fasteners	4.81	2.88	7/8 in. diam A325 threads excluded fasteners
Vertical Shear	3.60	2.16	$\Omega = 0.88$ with splice plates included
Horizontal Shear	1.45	0.87	
Partial Shear Yield	0.49	0.29	**Controls**
Whitmore Compression	1.04	0.62	
Tension	6.76	4.05	
Block Shear	3.99	2.39	
Chord Splice	60.0	36.0	

When the Partial Shear Plane Yield and/or Whitmore Compression capacity checks control and indicate a less than acceptable rating, more rigorous evaluation should be performed.

The following more rigorous rating checks are performed in Example 1:
(1) Horizontal shear capacity - Ω calculated: Supercedes Horizontal Shear with $\Omega = 0.88$.
(2) Basic Corner Check capacity (BCC): Replaces Partial Shear Plane Yield and Whitmore Compression capacity che
(3) Refined Corner Check capacity (RCC): Supercedes BCC unless BCC indicates acceptable rating.

G6.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω):

Global shear check along horizontal plane parallel with bottom chord. Shear force calculated using horizontal component of diagonal member forces. Gross section selected at bottom fastener of diagonal members to achieve maximum eccentricity. Net section calculated through bottom chord fastener holes. Ω calculated using Drucker formula.

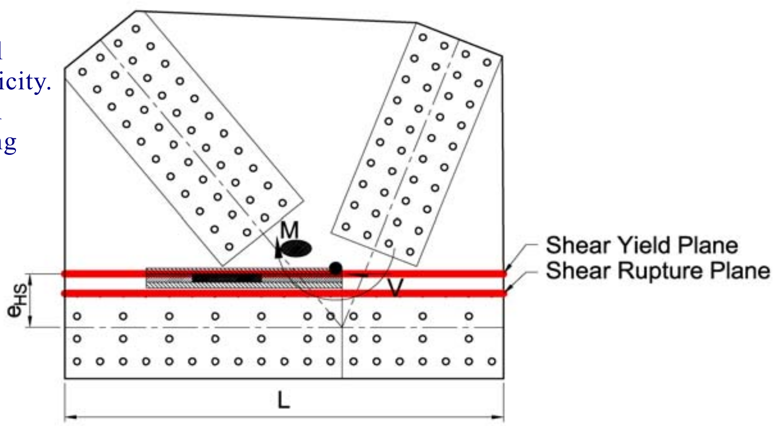


Figure 6: Horizontal Shear Between Web and Chord Members

$L = 57.1\,\text{in}$

$e_{HS} = 7.1\,\text{in}$

$M = V \cdot e_{HS}$

$d_h = 1\,\text{in}$

$n_{hole} = 19$

$$A_n = t \cdot \left(L - n_{hole} \cdot d_h\right) = \frac{1}{2}\text{in} \cdot [57.1\text{in} - (19) \cdot 1.0\text{in}] = 19.1\text{in}^2$$

G6.2.1a1 Gusset Plate Effective Thickness:

To account for strain hardening of the material, determine an effective thickness based on the proportion of the material ultimate strength to yield strength and limited by the actual thickness of the adjacent plate. This effectively bases the capacity calculation on F_u instead of F_y. Note that the deterioration is a "narrow band."

Actual material thickness along line of interest

$t_{mid.BB} = 0.25\,\text{in}$

Material thickness adjacent to line of interest

$t_{adj.BB} = 0.35\,\text{in}$

Effective material thickness when considering strain hardening

$$t_{eff.BB} = \min\left(t_{mid.BB} \cdot \frac{F_u}{F_y}, t_{adj.BB}\right) = \min\left(0.25\text{in} \cdot \frac{80\text{ksi}}{53\text{ksi}}, 0.35\text{in}\right) = \min(0.38\text{in}, 0.35\text{in}) = 0.35\text{in}$$

$t_{mid.DD} = 0.25\,\text{in}$

$t_{adj.DD} = 0.35\,\text{in}$

$$t_{eff.DD} = \min\left(t_{mid.DD} \cdot \frac{F_u}{F_y}, t_{adj.DD}\right) = \left(\min\left(0.25\text{in} \cdot \frac{80\text{ksi}}{53\text{ksi}}, 0.35\text{in}\right) = \min(0.38\text{in}, 0.35\text{in}) = 0.35\text{in}\right)$$

G6.2.1a2 Deteriorated Gusset Plate Section Properties:

G6.2.1a2.1 Gross Area:

$$A_g = L_{AA} \cdot t_{AA} + L_{BB} \cdot t_{eff.BB} + L_{CC} \cdot t_{CC} + L_{DD} \cdot t_{eff.DD} + L_{EE} \cdot t_{EE}$$

$$A_g = \left(10.6in \cdot \frac{1}{2}in + 6in \cdot 0.35in + 9in \cdot 0in + 10.5in\right) \cdot 0.35in + 21.0in \cdot \frac{1}{2}in = 21.6in^2$$

G6.2.1a2.1 Plastic Moment Capacity:

Assume that the plastic neutral axis lies within rightmost deteriorated zone (Section DD)

$$A_{PNA.left} = L_{AA} \cdot t_{AA} + L_{BB} \cdot t_{eff.BB} + L_{CC} \cdot t_{CC} + \left(L_{DD} - y\right) \cdot t_{eff.DD}$$

$$A_{PNA.right} = L_{EE} \cdot t_{EE} + y \cdot t_{eff.DD}$$

Set areas to the left and right of the plastic neutral axis equal to one another and solve for location of the plastic neutral axis with respect to the right edge of gusset

$$L_{AA} \cdot t_{AA} + L_{BB} \cdot t_{eff.BB} + L_{CC} \cdot t_{CC} + \left[L_{DD} - \left(y_{PNAr} - L_{EE}\right)\right] \cdot t_{eff.DD} = L_{EE} \cdot t_{EE} + \left(y_{PNAr} - L_{EE}\right) \cdot t_{eff.DD}$$

$$\left(L_{AA} \cdot t_{AA} + L_{BB} \cdot t_{eff.BB} + L_{CC} \cdot t_{CC} + L_{DD} \cdot t_{eff.DD} + L_{EE} \cdot t_{eff.DD}\right) - L_{EE} \cdot \left(t_{EE} - t_{eff.DD}\right) = 2y_{PNAr} \cdot t_{eff.DD}$$

$$y_{PNAr} = \frac{\left(L_{AA} \cdot t_{AA} + L_{BB} \cdot t_{eff.BB} + L_{CC} \cdot t_{CC} + L_{DD} \cdot t_{eff.DD} + L_{EE} \cdot t_{eff.DD}\right) - L_{EE} \cdot \left(t_{EE} - t_{eff.DD}\right)}{2 \cdot t_{eff.DD}}$$

$$y_{PNAr} = \frac{\left(10.6in \cdot \frac{1}{2}in + 6in \cdot 0.35in + 9in \cdot 0in + 10.5in \cdot 0.35in + 21.0in \cdot 0.35in\right) - 21.0in \cdot \left(\frac{1}{2}in - 0.35in\right)}{2 \cdot 0.35in} = 21.8in$$

Determine distance between centroids of left and right areas

$$y_{bar.left} = \frac{\left(L_{AA} \cdot t_{AA}\right) \cdot \left(L - y_{PNAr} - \frac{L_{AA}}{2}\right) + \left(L_{BB} \cdot t_{eff.BB}\right) \cdot \left(L - y_{PNAr} - L_{AA} - \frac{L_{BB}}{2}\right) \dots + \left(L_{CC} \cdot t_{CC}\right) \cdot \left(L - y_{PNAr} - L_{AA} - L_{BB} - \frac{L_{CC}}{2}\right) + \left(L_{DD} \cdot t_{eff.DD}\right) \cdot \left(\frac{L - y_{PNAr} - L_{AA} - L_{BB} - L_{CC}}{2}\right)}{L_{AA} \cdot t_{AA} + L_{BB} \cdot t_{eff.BB} + L_{CC} \cdot t_{CC} + \left[L_{DD} - \left(y_{PNAr} - L_{EE}\right)\right] \cdot t_{eff.DD}}$$

$$y_{bar.left} = \frac{\left(10.6in \cdot \frac{1}{2}in\right) \cdot \left(57.1in - 21.8in - \frac{10.6in}{2}\right) \dots + (6in \cdot 0.35in) \cdot \left(57.1in - 21.8in - 10.6in - \frac{6in}{2}\right) \dots + 0in^3 + (10.5in \cdot 0.35in) \cdot \left(\frac{57.1in - 21.8in - 10.6in - 6in - 9in}{2}\right)}{10.6in \cdot \frac{1}{2}in + 6in \cdot 0.35in + 0in^2 + [10.5in - (21.8in - 21.0in)] \cdot 0.35in} = 20.6in$$

G6.2.1a2.1 Plastic Moment Capacity Cont.:

$$y_{bar.right} = \frac{(L_{EE} \cdot t_{EE}) \cdot \left(y_{PNAr} - \frac{L_{EE}}{2}\right) + \left[\left(y_{PNAr} - L_{EE}\right) \cdot t_{eff.DD}\right] \cdot \left(\frac{y_{PNAr} - L_{EE}}{2}\right)}{L_{EE} \cdot t_{EE} + \left(y_{PNAr} - L_{EE}\right) \cdot t_{eff.DD}}$$

$$y_{bar.right} = \frac{\left(21.0\text{in} \cdot \frac{1}{2}\text{in}\right) \cdot \left(21.8\text{in} - \frac{21.0\text{in}}{2}\right) + [(21.8\text{in} - 21.0\text{in}) \cdot 0.35\text{in}] \cdot \left(\frac{21.8\text{in} - 21.0\text{in}}{2}\right)}{21.0\text{in} \cdot \frac{1}{2}\text{in} + (21.8\text{in} - 21.0\text{in}) \cdot 0.35\text{in}} = 11.0\text{in}$$

$$a = y_{bar.left} + y_{bar.right} = 20.6\text{in} + 11.0\text{in} = 31.6\text{in}$$

$$M_P = \left(A_g \cdot \frac{a}{2}\right) \cdot F_y = \left(21.6\text{in} \cdot \frac{31.6\text{in}}{2}\right) \cdot 53\text{ksi} = 18100\text{kip} \cdot \text{in}$$

G6.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω) Cont.:

Calculate Ω using Drucker formula instead of using AASHTO-specified Ω=0.88

$$V = V_p \cdot \left[1 - \left(\frac{M}{M_p}\right)\right]^{0.25} \qquad V = \Omega \cdot V_p$$

Drucker Formula [1]

$$V_P = (0.58) \cdot F_y \cdot A_g = (0.58) \cdot 53\text{ksi} \cdot 21.6\text{in}^2 = 663\text{kip}$$

Substitute $V = \Omega * V_p$ into Drucker formula and rearrange to solve for Ω using plastic shear and moment capacities

$$\Omega \cdot V_p = V_p \cdot \left(1 - \frac{\Omega \cdot V_p \cdot e_{HS}}{M_p}\right)^{0.25}$$

$$\Omega = \left(1 - \frac{\Omega \cdot V_P \cdot e_{HS}}{M_P}\right)^{0.25} = \left(1 - \frac{\Omega \cdot 663\text{kip} \cdot 7.1\text{in}}{18100\text{in} \cdot \text{kip}}\right)^{0.25} = 0.93$$

Requires iterative process since V is proportional to Ω. Can substitute AASHTO specified value of Ω = 0.88 on right side of equation as a first estimate of Ω. Result shown is the calculated value of Ω after performing necessary iterations.

$$\phi_{vy} = 1.0$$

$$\phi_{vu} = 0.85$$

$$C_Y = \varphi_{yy} \cdot (0.58) \cdot F_y \cdot A_g \cdot \Omega = 1.00(0.58) \cdot 53\text{ksi} \cdot 21.6\text{in}^2 \cdot (0.93) = 619\text{kip}$$

$$C_U = \varphi_{yu} \cdot (0.58) \cdot F_u \cdot A_n = 0.85(0.58) \cdot 80\text{ksi} \cdot 19.1\text{in}^2 = 752\text{kip}$$

$$C_{HS} = \min\left(C_Y, C_U\right) = \min(619\text{kip}, 752\text{kip}) = 619\text{kip}$$

Horizontal Shear Capacity (per plate)

G6.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω) Cont.:

Determine capacity of member M2 based on Horizontal Shear

$$C_{HS.M2} = C_{HS} \cdot \left| \frac{OpForce_{M2}}{OpForce_{M1} - OpForce_{M5}} \right| = 619kip \cdot \left| \frac{-1050kip}{548kip - (-443kip)} \right| = 656kip$$

Total member capacity

$$2 \cdot 656kip = 1312kip$$

$$ORF_{HS} = \frac{C_{HS.M2} - \gamma_{DL} \cdot \left| \frac{1}{2} DL_{M2} \right|}{\gamma_{LL} \cdot \left| \frac{1}{2} LL_{M2} \right|} = \frac{656kip - 1.3 \cdot \left| \frac{1}{2} \cdot -646kip \right|}{1.3 \cdot \left| \frac{1}{2} \cdot -162kip \right|} = 2.25$$

$$IRF_{HS} = \frac{C_{HS.M2} - \gamma_{DL} \cdot \left| \frac{1}{2} DL_{M2} \right|}{\gamma_{Inv.LL} \cdot \left| \frac{1}{2} LL_{M2} \right|} = \frac{656kip - 1.3 \cdot \left| \frac{1}{2} \cdot -646kip \right|}{2.17 \cdot \left| \frac{1}{2} \cdot -162kip \right|} = 1.35$$

[1] Drucker, D., *The Effect of Shear on the Plastic Bending of Beams*, American Society of Mechanical Engineers, NAMD Conference, Urbana, IL, June 1956

G6.2.2 Basic Corner Check:

The Basic Corner Check is a first-principles analytical approach utilizing fundamental steel design theory to conservatively calculate gusset plate limit state capacities at critical cross sections. This check is used to evaluate equilibrium and stability of a gusset plate "corner" bounded by horizontal and vertical planes that create the smallest section encompassing all fasteners of the diagonal member. The diagonal member force is assumed to be resisted by a combination of shear and normal forces acting on the vertical and horizontal surfaces bounding the "corner". Von Mises stress calculated on the surfaces is limited to the yield strength of the gusset plate. For simplicity and to avoid bending in the members, the resultant of each surface must pass through the work point. The "corner" will be adjusted in terms of location and plate thickness to accommodate deterioration.

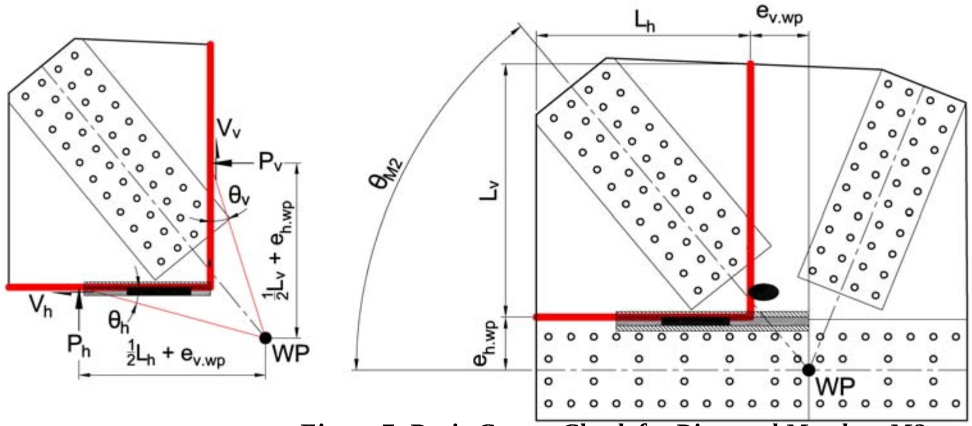


Figure 7: Basic Corner Check for Diagonal Member M2

Calculate resultant angles from the work point

$L_h = 28.4\,\text{in}$ $\qquad e_{h.wp} = 7.1\,\text{in}$

$L_v = 34\,\text{in}$ $\qquad e_{v.wp} = 7.8\,\text{in}$

G6.2.2a Horizontal Surface Check:

Since $L_h < L_v$ set (and there is more deterioration on the horizontal surface than the vertical surface) von Mises stress on horizontal surface equal to plate yield strength. After stresses on both surfaces are determined, verify assumption that horizontal surface is critical (i.e. reaches von Mises yield before vertical surface).

G6.2.2a1 Deteriorated Gusset Plate Section Properties:

Determine area and location of centroid for horizontal surface, considering deterioration

$$A_h = L_{AA} \cdot t_{AA} + L_{BB} \cdot t_{eff.BB} + L_{CC} \cdot t_{CC} + \left(L_h - L_{AA} - L_{BB} - L_{CC}\right) \cdot t_{eff.DD}$$

$$A_h = 10.6in \cdot \frac{1}{2} in + 6in \cdot 0.35in + 0in \cdot 9in + (28.4in - 10.6in - 6in - 9in) \cdot 0.35in = 8.37in^2$$

$$y_{bar.left} = \frac{\left(L_{AA} \cdot t_{AA}\right) \cdot \frac{L_{AA}}{2} + \left(L_{BB} \cdot t_{eff.BB}\right) \cdot \left(L_{AA} + \frac{L_{BB}}{2}\right) + \left(L_{CC} \cdot t_{CC}\right) \cdot \left(L_{AA} + L_{BB} + \frac{L_{CC}}{2}\right) \ldots + \left[\left(L_h - L_{AA} - L_{BB} - L_{CC}\right) \cdot t_{eff.DD}\right] \cdot \left[L_{AA} + L_{BB} + L_{CC} + \frac{\left(L_h - L_{AA} - L_{BB} - L_{CC}\right)}{2}\right]}{L_{AA} \cdot t_{AA} + L_{BB} \cdot t_{eff.BB} + L_{CC} \cdot t_{CC} + \left(L_h - L_{AA} - L_{BB} - L_{CC}\right) \cdot t_{eff.DD}}$$

$$y_{bar.left} = \frac{\left(10.6in \cdot \frac{1}{2} in\right) \cdot \frac{10.6in}{2} + (6in \cdot 0.35in) \cdot \left(10.6in + \frac{6in}{2}\right) + (9in \cdot 0in) \cdot \left(10.6in + 6in + \frac{9in}{2}\right) \ldots + [(28.4in - 10.6in - 6in - 9in) \cdot 0.35in] \cdot \left[10.6in + 6in + 9in + \frac{(28.4in - 10.6in - 6in - 9in)}{2}\right]}{10.6in \cdot \frac{1}{2} in + 6in \cdot 0.35in + 9in \cdot 0in + (38.4in - 10.6in - 6in - 9in) \cdot 0.35in}$$

$$y_{bar.left} = 9.9\,in$$

G6.2.2a2 Determine Horizontal Surface Resultants:

$$\theta_h = atan\left(\frac{e_{h.wp}}{L_h + e_{v.wp} - y_{bar.left}}\right) = atan\left(\frac{7.1in}{28.4in + 7.8in - 9.9in}\right) = 15.2deg$$

$$P_h = V_h \cdot tan\left(\theta_h\right)$$

$$\sigma_h = \frac{P_h}{A_h} = \frac{P_h}{L_h \cdot t}$$

$$\tau_h = \frac{V_h}{A_h} = \frac{V_h}{L_h \cdot t}$$

$$\sigma_{vm} = \sqrt{\sigma_h^2 + 3\tau_h^2}$$

G6.2.2a2 Determine Horizontal Surface Resultants Cont.:

Substitute P_h as a function of V_h and set the von Mises stress to yield

$$F_y = 53\text{ksi} = \sigma_{vm} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}} = \sqrt{\left(\frac{P_h}{A_h}\right)^2 + 3\cdot\left(\frac{V_h}{A_h}\right)^2} = \sqrt{\left(\frac{V_h\cdot\tan(15.2\text{deg})}{8.37\text{in}^2}\right)^2 + 3\cdot\left(\frac{V_h}{8.37\text{in}^2}\right)^2}$$

Rearrange terms and solve for V_h

$$V_h = \frac{A_h\cdot F_y}{\sqrt{\tan(\theta_h)^2 + 3}} = \frac{8.37\text{in}^2\cdot 53\text{ksi}}{\sqrt{\tan(15.2\text{deg})^2 + 3}} = 253\text{kip}$$

Solve for P_h

$$P_h = V_h\cdot\tan(\theta_h) = 253\text{kip}\cdot\tan(15.2\text{deg}) = 69\text{kip}$$

Calculate shear and normal stresses on horizontal surface

$$\sigma_h = \frac{P_h}{A_h} = \frac{69\text{kip}}{8.37\text{in}^2} = 8.2\text{ksi} \qquad \tau_h = \frac{V_h}{A_h} = \frac{253\text{kip}}{8.37\text{in}^2} = 30.2\text{ksi}$$

G6.2.2b Vertical Surface Check:

Determine forces and stresses on vertical surface based on horizontal surface forces and stated constraints (i.e. force resultants to pass thru workpoint).

G6.2.2b1 Deteriorated Gusset Plate Section Properties:

Determine area and location of centroid while considering small amount of deterioration at bottom of surface.

$$A_v = L_{WW}\cdot t_{WW} + L_{XX}\cdot t_{XX} + L_{YY}\cdot t_{YY} + L_{ZZ}\cdot t_{ZZ} = 30.1\text{in}\cdot\frac{1}{2}\text{in} + 1.1\text{in}\cdot 0\text{in} + 2.1\text{in}\cdot\frac{1}{2}\text{in} + 0.75\text{in}\cdot 0.35\text{in} = 16.4\text{in}^2$$

$$y_{bar.v} = \frac{\begin{bmatrix}(L_{WW}\cdot t_{WW})\cdot\left(L_{ZZ} + L_{YY} + L_{XX} + \frac{L_{WW}}{2}\right) + (L_{XX}\cdot t_{XX})\cdot\left(L_{ZZ} + L_{YY} + \frac{L_{XX}}{2}\right) \ldots \\ + (L_{YY}\cdot t_{YY})\cdot\left(L_{ZZ} + \frac{L_{YY}}{2}\right) + (L_{ZZ}\cdot t_{ZZ})\cdot\left(\frac{L_{ZZ}}{2}\right)\end{bmatrix}}{A_v}$$

$$y_{bar.v} = \frac{\begin{matrix}\left(30.1\text{in}\cdot\frac{1}{2}\text{in}\right)\cdot\left(0.75\text{in} + 2.1\text{in} + 1.1\text{in} + \frac{30.1\text{in}}{2}\right) + (1.09\text{in}\cdot 0\text{in})\cdot\left(0.75\text{in} + 2.1\text{in} + \frac{1.1\text{in}}{2}\right) \ldots \\ + \left(2.1\text{in}\cdot\frac{1}{2}\text{in}\right)\cdot\left(0.75\text{in} + \frac{2.1\text{in}}{2}\right) + (0.75\text{in}\cdot 0.35\text{in})\cdot\left(\frac{0.75\text{in}}{2}\right)\end{matrix}}{16.4\text{in}^2}$$

$$y_{bar.v} = 17.6\text{in}$$

G6.2.2b2 Determine Vertical Surface Resultants:

$$\theta_v = \operatorname{atan}\left(\frac{e_{v.wp}}{e_{h.wp} + y_{bar.v}}\right) = \operatorname{atan}\left(\frac{7.8in}{7.1in + 17.6in}\right) = 17.4deg$$

$$P_v = V_v \cdot \tan(\theta_v)$$

Substitute P_v as a function of V_v

$$\theta_{M2} = \operatorname{atan}\left(\frac{V_v + P_h}{P_v + V_h}\right) = \operatorname{atan}\left(\frac{V_v + P_h}{V_v \cdot \tan(\theta_v) + V_h}\right) = \operatorname{atan}\left(\frac{V_v + 69kip}{V_v \cdot \tan(17.4deg) + 253kip}\right)$$

Rearrange terms and solve for V_v. Substitute values obtained from previously solving P_h and V_h.

$$V_v = \frac{P_h - V_h \cdot \tan(\theta_{M2})}{\tan(\theta_{M2}) \cdot \tan(\theta_v) - 1} = \frac{69kip - 253kip \cdot \tan(50.5deg)}{\tan(50.5deg) \cdot \tan(17.4deg) - 1} = 384kip$$

Solve for P_v

$$P_v = V_v \cdot \tan(\theta_v) = 384kip \cdot \tan(17.4deg) = 120kip$$

Calculate shear and normal stresses on vertical surface

$$\sigma_v = \frac{P_v}{A_v} = \frac{120kip}{16.4in^2} = 7.4ksi \qquad \tau_v = \frac{V_v}{A_v} = \frac{384kip}{16.4in^2} = 23.5ksi$$

Calculate von Mises stress

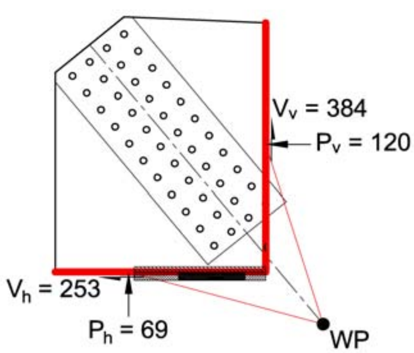
$$\sigma_{vm.v} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{(7.4ksi)^2 + 3 \cdot (23.5ksi)^2} = 41.3ksi \quad \le \quad F_y = 53ksi$$

Since von Mises stress on vertical surface is less than yield strength of the gusset plate, the horizontal surface controls. If this had not been the case, the von Mises stress calculated on the vertical surface would have been greater than the yield stress. The previous process would have been modified by first setting the von Mises stress on the vertical surface to the yield stress and then determining the necessary resultants on the horizontal surface to balance the moment about the work point.

Substitute corresponding solved forces to determine member resultant force.

$$C_{BCC.vM} = \sqrt{(V_h + P_v)^2 + (V_v + P_h)^2} = \sqrt{(253kip + 120kip)^2 + (384kip + 69kip)^2} = 587kip$$

BCC von Mises Capacity (per plate)



Total member capacity
$2 \cdot 586kip = 1173kip$

Figure 8: Basic Corner Check Resultants for Diagonal Member M2

G6.2.2c BCC Buckling Check:

Check plate buckling due to axial forces on Basic Corner Check surfaces (refer to Appendix B). If buckling controls, then von Mises stresses must be adjusted.

As a first pass, do not consider the distance a_h extending from the horizontal surface of the corner check shown below which runs through the deterioration, but extending from the horizontal surface determined by the typical, undeteriorated corner check. This would represent a worst-case buckling condition and may circumvent checking both corner possibilities. If the buckling associated with a_h controls, this can be refined if warranted.

Evaluate buckling capacity of plate regions defined by the short and long gap between the Basic Corner Check surfaces and adjacent member connections.

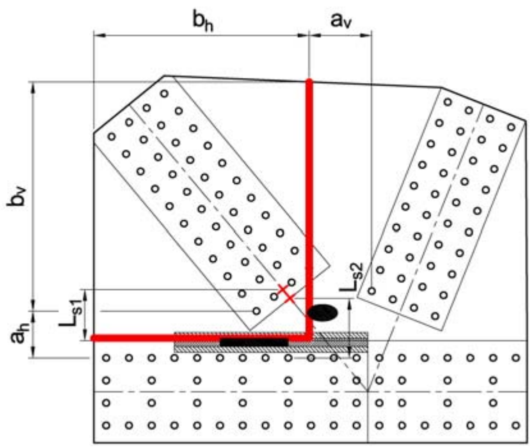


Figure 9: Corner Check Buckling Lengths

G6.2.2c1 Short Gap Buckling Check:

For this gusset plate, the short gap corresponds to the horizontal surface ($a_h < a_v$). a_h and a_v are defined as the distances from the respective Corner Check surface to the parallel line passing through the nearest fastener in an adjacent member.

Determine effective moment of inertia for column

$$L_s = \frac{L_{s1} + L_{s2}}{2} = \frac{6.8\text{in} + 7.9\text{in}}{2} = 7.4\text{in}$$

$$t_1 = t = \frac{1}{2}\,\text{in}$$

$$t_{2.1} = t = \frac{1}{2}\,\text{in}$$

$$t_{2.2} = 0.35\text{in}$$

$$t_{3.1} = t = \frac{1}{2}\,\text{in}$$

$$t_{3.2} = 0.25\text{in}$$

$$t_{4.1} = t = \frac{1}{2}\,\text{in}$$

$$t_{4.2} = 0.35\text{in}$$

$$t_5 = t = \frac{1}{2}\,\text{in}$$

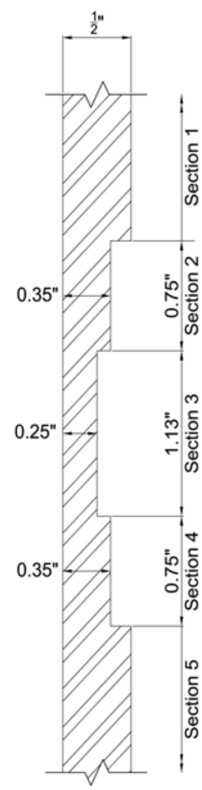


Figure 10: Short Gap Buckling Section View

G6.2.2c1 Short Gap Buckling Check Cont.:

Determine moment of inertia for different sections (1 through 5) along buckling length.

$$I_1 = \frac{L_h \cdot t_1^{\ 3}}{12} = \frac{28.4\,\text{in} \cdot \left(\frac{1}{2}\text{in}\right)^3}{12} = 0.295\,\text{in}^4$$

$$I_2 = \frac{L_{AA} \cdot t_{2.1}^{\ 3}}{12} + \frac{\left(L_h - L_{AA}\right) \cdot t_{2.2}^{\ 3}}{12} = \frac{10.6\text{in} \cdot \left(\frac{1}{2}\text{in}\right)^3}{12} + \frac{(28.4\text{in} - 10.6\text{in}) \cdot (0.35\text{in})^3}{12} = 0.174\,\text{in}^4$$

$$I_3 = \frac{L_{AA} \cdot t_{3.1}^{\ 3}}{12} + \frac{L_{BB} \cdot t_{3.2}^{\ 3}}{12} + \frac{\left(L_h - L_{AA} - L_{BB} - L_{CC}\right) \cdot t_{3.2}^{\ 3}}{12}$$

$$I_3 = \frac{10.6\text{in} \cdot \left(\frac{1}{2}\text{in}\right)^3}{12} + \frac{6\text{in} \cdot (0.25\text{in})^3}{12} + \frac{(28.4\text{in} - 10.6\text{in} - 6\text{in} - 9\text{in}) \cdot (0.25\text{in})^3}{12} = 0.122\,\text{in}^4$$

$$I_4 = \frac{L_{AA} \cdot t_{4.1}^{\ 3}}{12} + \frac{\left(L_h - L_{AA}\right) \cdot t_{4.2}^{\ 3}}{12} = \frac{10.6\text{in} \cdot \left(\frac{1}{2}\text{in}\right)^3}{12} + \frac{(28.4\text{in} - 10.6\text{in}) \cdot (0.35\text{in})^3}{12} = 0.174\,\text{in}^4$$

$$I_5 = \frac{L_h \cdot t_5^{\ 3}}{12} = \frac{28.4\text{in} \cdot \left(\frac{1}{2}\text{in}\right)^3}{12} = 0.295\,\text{in}^4$$

To determine the effective moment of inertia, a lateral load of 1 kip was allowed to act on the actual section (comprised of the moments of inertia 1 through 5 from above). The analysis, which was performed separately, showed this load to cause a deflection of 0.0059".

$$\Delta_{1kip} = 0.0059\text{in}$$

$$\Delta_{1kip} = \frac{1\text{kip} \cdot L_s^{\ 3}}{12 \cdot E \cdot I_o}$$

$$I_o = \frac{1\text{kip} \cdot L_s^{\ 3}}{12 \cdot E \cdot \Delta_{1kip}} = \frac{1\text{kip} \cdot (7.4\text{in})^3}{12 \cdot 29000\text{ksi} \cdot 0.0059\text{in}} = 0.194\,\text{in}^4$$

Determine effective buckling thickness

$$I_o = \frac{L_h \cdot t_{eff}^{\ 3}}{12}$$

$$t_{eff} = \left(\frac{I_o \cdot 12}{L_h}\right)^{\frac{1}{3}} = \left(\frac{0.194\text{in}^4 \cdot 12}{28.4\text{in}}\right)^{\frac{1}{3}} = 0.43\text{in}$$

G6.2.2c1 Short Gap Buckling Check Cont.:

$$A_{eff} = t_{eff} \cdot L_h = 0.43in \cdot 28.4in = 12.3in^2$$

$$r_{eff} = \frac{t_{eff}}{\sqrt{12}} = 0.13in$$

$$F_e = \frac{\pi^2 \cdot E}{\left(\frac{K \cdot L_s}{r}\right)^2} = \frac{\pi^2 \cdot 29000ksi}{\left[\frac{1.0 \cdot (7.4in)}{0.13in}\right]^2} = 83.2ksi$$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2 \cdot \sqrt{2}}\right) = 53ksi \cdot \left(1 - \frac{\sqrt{\frac{53ksi}{83.2ksi}}}{2 \cdot \sqrt{2}}\right) = 38.0ksi$$

$$\sigma = \frac{P_h}{A_{eff}} = \frac{69kip}{12.3in^2} = 5.6ksi$$

$$\tau = \frac{V_h}{A_{eff}} = \frac{253kip}{12.3in^2} = 20.5ksi$$

$$\sigma_{Principle} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{5.6ksi}{2} + \sqrt{\left(\frac{5.6ksi}{2}\right)^2 + (20.5ksi)^2} = 23.5ksi \qquad \leq \qquad F_{cr} = 38.0\,ksi$$

The principle stress is less than the critical buckling stress; therefore, buckling is not a concern

G6.2.2c2 Long Gap Buckling Check:

Treat long gap as flat rectangular plate with one non-loaded edge fixed and the remaining edges clamped (dashed curve D in Figure 11)

Long Gap Length (Figure 11)

$$a = a_v = 8.25\text{in}$$

Length of Long Side Surface (Figure 11)

$$b = L_v = 34.0\text{in}$$

$$\frac{a}{b} = \frac{8.25\text{in}}{34.0\text{in}} = 0.24$$

Because a/b is less than 0.75 (where k curve is nearly asymptotic), buckling of long gap plate is not a concern. Otherwise calculate k as follows (using an approximate best fit function of dashed curve D in Figure 11):

$$k = 4.64 \cdot \left(\frac{a}{b}\right)^{-1.106}$$

$$F_e = \frac{k \cdot \pi^2 \cdot E}{12\left(1 - \nu^2\right) \cdot \left(\frac{b}{t}\right)^2}$$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2 \cdot \sqrt{2}}\right)$$

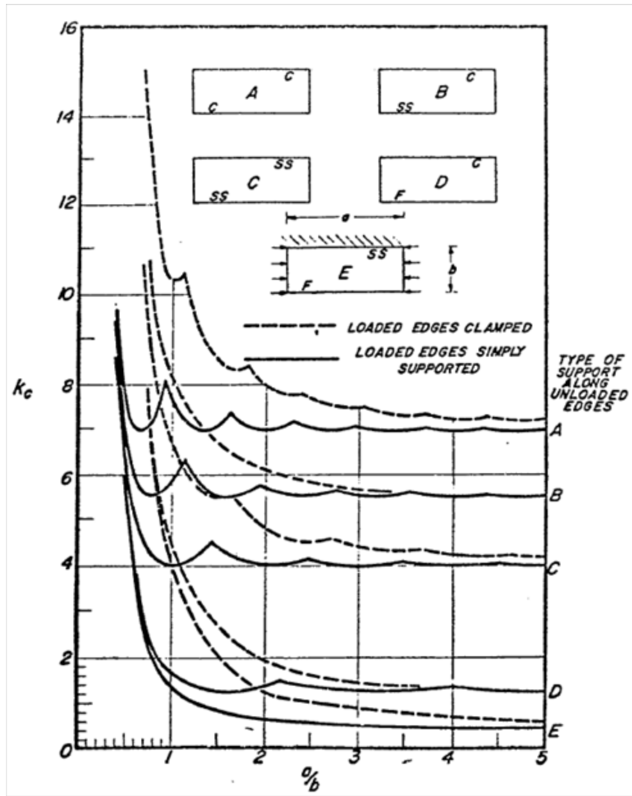


Figure 11: Elastic Buckling Coefficients [2]

Compare calculated principle stress to critical stress.

$$\sigma_{Princ} = \frac{\sigma_v}{2} + \sqrt{\left(\frac{\sigma_v}{2}\right)^2 + \tau_v^2} \quad \le \quad F_{cr}$$

G6.2.2 Basic Corner Check Cont.:

Since buckling of the short and long gaps are not a concern for the Basic Corner Check, no reduction in calculated capacity is required, and capacity calculated using von Mises stress applies.

$C_{BCC} = 586\text{kip}$

BCC Resultant Capacity (per plate)

Total member capacity
$2 \cdot 586\text{kip} = 1173\text{kip}$

$$ORF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{586\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -646\text{kip}\right|}{1.3 \cdot \left|\frac{1}{2} \cdot -162\text{kip}\right|} = 1.59$$

$$IRF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{586\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -646\text{kip}\right|}{2.17 \cdot \left|\frac{1}{2} \cdot -161\text{kip}\right|} = 0.95$$

If an increased rating factor is required, perform a Refined Corner Check.

[2] George Gerard and Herbert Becker. *Handbook of Structural Stability*, Part I - Buckling of Flat Plates, Tech. Note 3871, National Advisory Committee for Aeronautics, Washington, D.C., July 1957.

G6.2.3 Refined Corner Check:

The Refined Corner Check removes the constraint that surface resultants pass through the work point as assumed in the Basic Corner Check. In removing this constraint, it is important to check the portion of gusset plate outside of the corner (Stub) and check again for plate buckling based on these resultants.

An efficient initial starting point in this iterative check is to force the resultants acting on each surface to be parallel t(the member and then adjust shear and normal forces as necessary.

G6.2.3a Horizontal Surface Check: Parallel Resultants

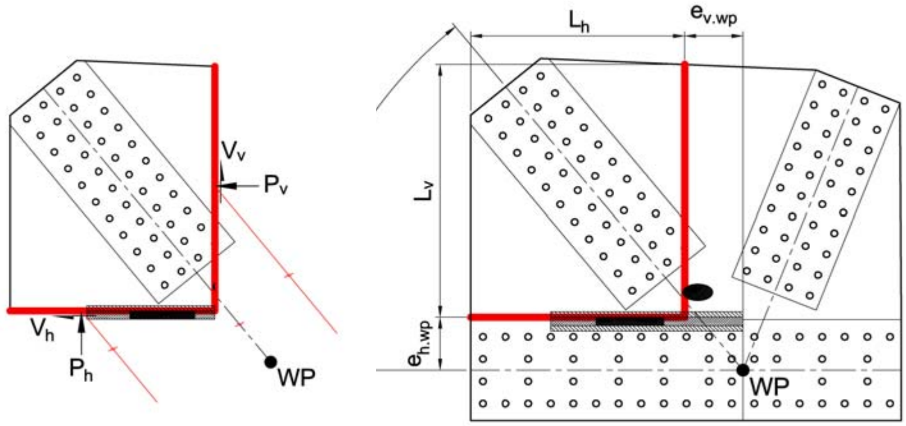


Figure 12: Refined Corner Check for Diagonal Member M2

As with the Basic Corner Check, check to see if the horizontal surface is the controlling surface by setting von Mises stress on horizontal surface equal to plate yield strength. After stresses on both surfaces are determined, verify assumption that horizontal surface is critical (i.e. reaches von Mises yield before vertical surface).

$$V_h = \frac{P_h}{\tan(\theta_{M2})}$$

Constrain von Mises on surface to the yield stress

$$\sigma_{vm} = \sqrt{\sigma_h^2 + 3\tau_h^2} = F_y$$

$$L_h = 28.4\,\text{in}$$

$$\theta_{M2} = 50.5\cdot\text{deg}$$

Substitute V_h as a function of P_h and set the von Mises stress to yield.

$$F_y = 53\text{ksi} = \sigma_{vm} = \sqrt{\sigma_h^2 + 3\tau_h^2} = \sqrt{\left(\frac{P_h}{A_h}\right)^2 + 3\cdot\left(\frac{V_h}{A_h}\right)^2} = \sqrt{\left(\frac{P_h}{8.37\text{in}^2}\right)^2 + 3\cdot\left(\frac{\frac{P_h}{\tan(\theta_{M2})}}{8.37\text{in}^2}\right)^2}$$

G6.2.3a Horizontal Surface Check Cont.: Parallel Resultants

Rearrange terms and solve for P_h

$$P_h = \frac{F_y \cdot A_h \cdot \tan(\theta_{M2})}{\sqrt{\tan(\theta_{M2})^2 + 3}} = \frac{53ksi \cdot 8.37in^2 \cdot \tan(50.5deg)}{\sqrt{\tan(50.5deg)^2 + 3}} = 254kip$$

Solve for V_h

$$V_h = \frac{P_h}{\tan(\theta_{M2})} = \frac{431kip}{\tan(50.5deg)} = 210kip$$

Calculate resultants stresses on horizontal surface

$$\sigma_h = \frac{P_h}{A_h} = \frac{254kip}{8.37in^2} = 30.3ksi \qquad \tau_h = \frac{V_h}{A_h} = \frac{210kip}{8.37in^2} = 25.1ksi$$

G6.2.3b Vertical Surface Check: Parallel Resultants

Constrain moments about work point to balance (i.e. $\Sigma M_{WP} = 0$)

$$V_v = P_v \cdot \tan(\theta_{M2})$$

$$L_v = 34.0\,in$$

$$e_{v.wp} = 7.8\,in$$

$$e_{h.wp} = 7.1\,in$$

$$y_{bar.v} = 17.6\,in$$

$$y_{bar.left} = 9.9\,in$$

$$\sum M = 0 = \left[P_h \cdot (L_h + e_{v.wp} - y_{bar.left}) - V_h \cdot e_{h.wp}\right] - \left[P_v \cdot (y_{bar.v} + e_{h.wp}) - V_v \cdot e_{v.wp}\right]$$

Substitute V_v as a function of P_v, rearrange terms and solve for P_v

$$0 = \left[P_h \cdot (L_h + e_{v.wp} - y_{bar.left}) - V_h \cdot e_{h.wp}\right] - \left[P_v \cdot (y_{bar.v} + e_{h.wp}) - P_v \cdot \tan(\theta_{M2}) \cdot e_{v.wp}\right]$$

$$P_v = \frac{\left[P_h \cdot (L_h + e_{v.wp} - y_{bar.left}) - V_h \cdot e_{h.wp}\right]}{\left[(y_{bar.v} + e_{h.wp}) - \tan(\theta_{M2}) \cdot e_{v.wp}\right]} = \frac{254kip \cdot (28.4in + 7.8in - 9.88in) - 210kip \cdot (7.1in)}{17.6in + 7.1in - \tan(50.5deg) \cdot 7.8in} = 337kip$$

Solve for V_v

$$V_v = P_v \cdot \tan(\theta_{M2}) = 338kip \cdot \tan(50.5deg) = 409kip$$

G6.2.3b Vertical Surface Check Cont.: Parallel Resultants

Calculate resultants stresses on vertical surface

$$\sigma_v = \frac{P_v}{A_v} = \frac{338\text{kip}}{16.4\text{in}^2} = 19.8\text{ksi} \qquad \tau_v = \frac{V_v}{A_v} = \frac{410\text{kip}}{16.4\text{in}^2} = 24.0\text{ksi}$$

Calculate von Mises stress on vertical surface

$$\sigma_{vm.v} = \sqrt{\sigma_v^{\,2} + 3\tau_v^{\,2}} = \sqrt{(19.8\text{ksi})^2 + 3\cdot(24.0\text{ksi})^2} = 46.1\text{ksi} \qquad \leq \; F_y = 53\,\text{ksi}$$

G6.2.3 Refined Corner Check Cont.: Parallel Resultants

Since the von Mises stress on the vertical surface is less than the yield strength of the gusset plate, the horizontal surface controls, as assumed. If this had not been the case, the von Mises stress calculated on the vertical surface would have been greater than the yield stress. The previous process would have been modified by first setting the von Mises stress on the vertical surface to the yield stress and then determining the necessary resultants on the horizontal surface to balance the moment about the work point.

$$C_{RCC} = \sqrt{\left(V_h + P_v\right)^2 + \left(V_v + P_h\right)^2} = \sqrt{(210\text{kip} + 338\text{kip})^2 + (410\text{kip} + 254\text{kip})^2} = 860\text{kip}$$

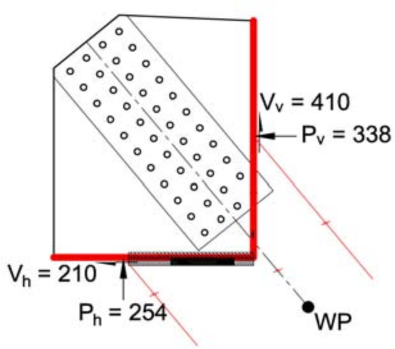


Figure 13: Refined Corner Check with Parallel Resultants to Member

G6.2.3c Remaining Portion (Stub) Check: Parallel Resultants

Determine equivalent concurrent forces for vertical and tension diagonal per plate

$$F_{RCC.M3} = OpForce_{M3} \cdot \left| \frac{C_{RCC}}{OpForce_{M2}} \right| = 893kip \cdot \left| \frac{860kip}{-1050kip} \right| = 731kip$$

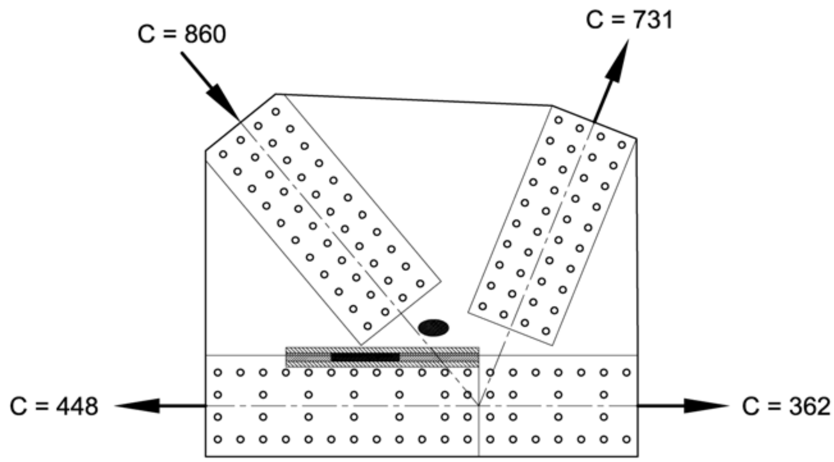


Figure 14: Concurrent Member Capacities (per plate) Based on Refined Corner Check (Subject to Stub Check and Buckling Check)

Check remaining portion of the gusset plate outside of the corner and chord. Select a Section Q that encompasses all forces applied by member M3.

$L_Q = 28.74$ in

$L_{SD} = 7.75$ in

$\theta_{M3} = 68.23 \cdot deg$

$L_{M3\_CC} = 10.6in$

$L_v = 34.0$ in

$e_{h.wp} = 7.1$ in

$e_{Q.wp} = e_{h.wp}$

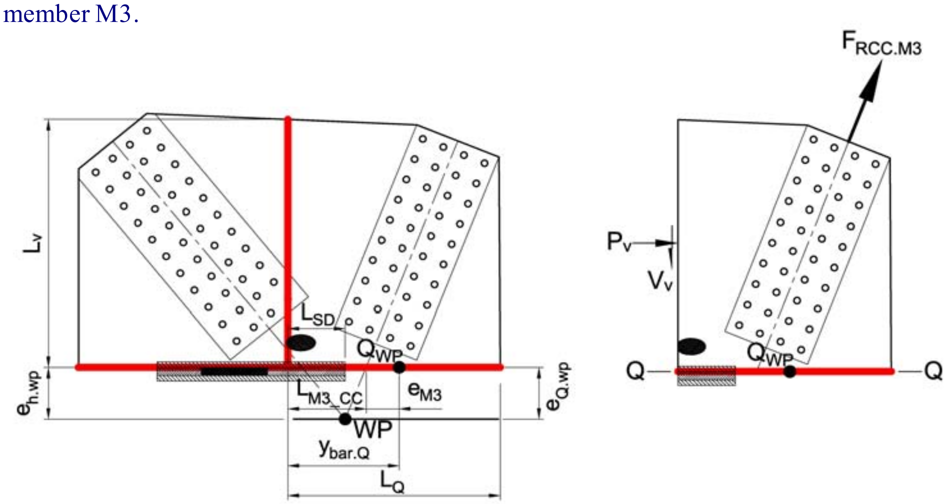


Figure 15: Remaining Gusset Plate Stub

Calculate forces P_Q and V_Q along Section Q

$$P_Q = F_{RCC.M4} \cdot \sin(\theta_{M3}) - V_v = 731kip \cdot \sin(68.2deg) - 410kip = 270kip$$

$$V_Q = F_{RCC.M4} \cdot \cos(\theta_{M3}) + P_v = 731kip \cdot \cos(68.2deg) + 338kip = 608kip$$

G6.2.3c Remaining Portion (Stub) Check Cont.: Parallel Resultants

Determine Section Properties along Section Q

$$A_Q = L_{SD} \cdot t_{eff.DD} + L_{EE} \cdot t_{EE} = 7.75in \cdot 0.25in + 21.0in \cdot \frac{1}{2} in = 13.2in^2$$

$$y_{bar.Q} = \frac{\left(L_{SD} \cdot t_{eff.DD}\right) \cdot \left(\frac{L_{SD}}{2}\right) + \left(L_{EE} \cdot t_{EE}\right) \cdot \left(L_{SD} + \frac{L_{EE}}{2}\right)}{A_Q}$$

$$y_{bar.Q} = \frac{(7.75in \cdot 0.35in) \cdot \left(\frac{7.75in}{2}\right) + \left(21.0in \cdot \frac{1}{2} in\right) \cdot \left(7.75in + \frac{21.0in}{2}\right)}{13.2in^2} = 15.1in$$

$$I_Q = \frac{t_{eff.DD} \cdot L_{SD}^3}{12} + \frac{t_{EE} \cdot L_{EE}^3}{12} + \left(L_{SD} \cdot t_{eff.DD}\right) \cdot \left(\frac{L_{SD}}{2} - y_{bar.Q}\right)^2 + \left(L_{SE} \cdot t_{EE}\right) \cdot \left(L_{SD} + \frac{L_{EE}}{2} - y_{bar.Q}\right)^2$$

$$I_Q = \frac{0.35in \cdot (7.75in)^3}{12} + \frac{\frac{1}{2} in \cdot (21.0in)^3}{12} \ldots + (7.75in \cdot 0.35in) \cdot \left(\frac{7.75in}{2} - 15.1in\right)^2 + \left(21.0in \cdot \frac{1}{2} in\right) \cdot \left(7.75in + \frac{21.0in}{2} - 15.1in\right)^2 = 745in^4$$

$$S_Q = \frac{I_Q}{\max\left(y_{bar.Q}, L_{SD} + L_{EE} - y_{bar.Q}\right)} = \frac{745in^4}{\max(15.1in, 7.75in + 21.0in - 15.1in)} = 49.4in^3$$

$$e_{M3} = y_{bar.Q} - L_{M3\_CC} = 4.5in$$

Calculate moment M_Q acting at Q_{WP}

$$M_Q = P_v \cdot \left(y_{bar.v}\right) - V_v \cdot y_{bar.Q} + F_{RCC.M3} \cdot \sin\left(\theta_{M3}\right) \cdot e_{M3}$$

$$M_Q = 338kip \cdot (17.6in) - 410kip \cdot 15.1in + 731kip \cdot \sin(68.2deg) \cdot 4.47in = 2805kip \cdot in$$

$$\sigma_P = \frac{P_Q}{A_Q} = \frac{270kip}{13.2in^2} = 20.4ksi$$

$$\sigma_M = \frac{\left|M_Q\right|}{S_Q} = \frac{|2805kip \cdot in|}{49.4in^3} = 57ksi$$

The peak normal stress from this combination of forces is greater than the yield stress. Reducing forces to limit the peak normal stress to be below the yield stress would reduce the calculated capacity based on the Refined Corner Check below that already calculated using the Basic Corner Check. Instead, remove the restriction that the resultants must be parallel and calculate a new capacity.

G6.2.4 Refined Corner Check: Nonparallel Resultants

Since the stub was overstressed with the initial combination of forces from the Refined Corner Check with the Parallel Resultants, aim to have a similar combination of forces based on the Basic Corner Check as a new starting point (rather than parallel resultants). Since the horizontal surface has controlled previously, constrain the von Mises stress along this surface to equal the yield stress. Increasing the axial stress on this surface is the only way to increase the overall capacity. This will also reduce the moment acting on the stub.

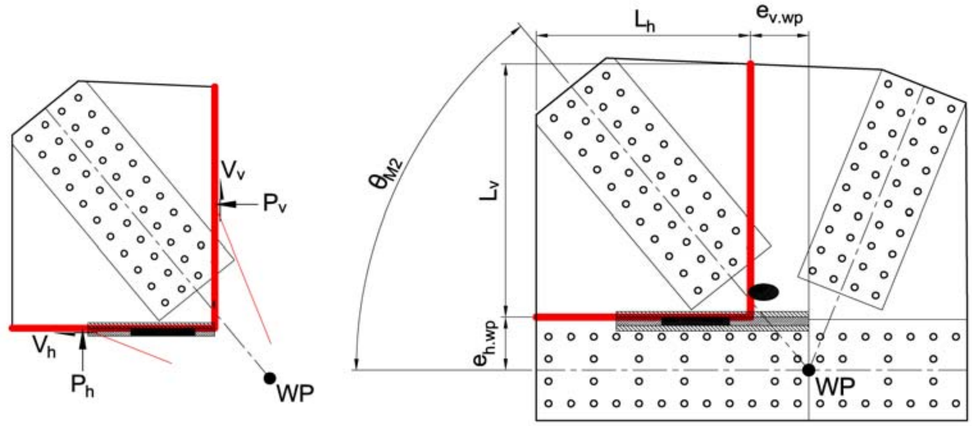


Figure 16: Refined Corner Check for Diagonal Member M2

G6.2.4a Determine Trial Forces and Overall Capacity with All Forces a Function of V_h:

G6.2.4a1 - Horizontal Surface

Solve the von Mises stress relationship for the axial force on the horizontal surface so that V_h is a function of P_h

$$F_y^2 = \sigma^2 + 3 \cdot \tau^2$$

$$F_y^2 = \left(\frac{P_h}{A_h}\right)^2 + 3 \cdot \left(\frac{V_h}{A_h}\right)^2$$

$$V_h = \sqrt{\frac{F_y^2 \cdot A_h^2 - P_h^2}{3}}$$

G6.2.4a2 - Vertical Surface

Solve for the forces acting on the vertical surface as a function of the forces acting on the horizontal surface.

Constrain final resultant to be parallel to member to avoid bending in member.

$$\operatorname{atan}\left(\frac{P_h + V_v}{V_h + P_v}\right) = \theta_{M2}$$

Constrain moments about work point to balance.

$$\sum M = 0 = \left[P_h \cdot \left(L_h + e_{v.wp} - y_{bar.left}\right) - V_h \cdot e_{h.wp}\right] - \left[P_v \cdot \left(y_{bar.v} + e_{h.wp}\right) - V_v \cdot e_{v.wp}\right]$$

G6.2.4a Determine Trial Forces and Overall Capacity with All Forces a Function of V_h Cont.:

Solve two equations for P_h and V_h

$$P_v = \frac{P_h + V_v}{\tan(\theta_{M2})} - V_h$$

$$V_v = \frac{P_v \cdot (y_{bar.v} + e_{h.wp}) + V_h \cdot e_{h.wp} - P_h \cdot (L_h + e_{v.wp} - y_{bar.left})}{e_{v.wp}}$$

$$P_v = \frac{P_h + \dfrac{P_v \cdot (y_{bar.v} + e_{h.wp}) + V_h \cdot e_{h.wp} - P_h \cdot (L_h + e_{v.wp} - y_{bar.left})}{e_{v.wp}}}{\tan(\theta_{M2})} - V_h$$

Substitute for P_v and V_v combine terms and simplify

$$P_v = \frac{P_h \cdot (L_h - y_{bar.left}) + V_h \cdot (e_{v.wp} \cdot \tan(\theta_{M2}) - e_{h.wp})}{e_{h.wp} + y_{bar.v} - e_{v.wp} \cdot \tan(\theta_{M2})}$$

G6.2.4a3 - Trial Force Substitution:

Choose a value for the axial force on the horizontal surface (P_h) that gives a calculated capacity at least that of Horizontal Shear.

Recall: $C_{HS.M2} = 656.14\,kip$ Therefore, select $P_h = 125\,kip$

Solve for the following:

$$V_h = \sqrt{\frac{F_y^2 \cdot A_h^2 - P_h^2}{3}} = \sqrt{\frac{(53ksi)^2 \cdot 8.37in^2 - (125kip)^2}{3}} = 246kip$$

$$P_v = \frac{P_h \cdot (L_h - y_{bar.left}) + V_h \cdot (e_{v.wp} \cdot \tan(\theta_{M2}) - e_{h.wp})}{e_{h.wp} + y_{bar.v} - e_{v.wp} \cdot \tan(\theta_{M2})}$$

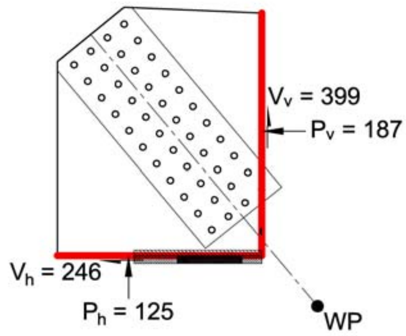
$$P_v = \frac{125kip \cdot (28.4in - 9.9in) + 246kip \cdot (7.8in \cdot \tan(50.5deg) - 7.1in)}{7.1in + 17.6in - 7.8in \cdot \tan(50.5deg)} = 187kip$$

$$V_v = \frac{P_v \cdot (y_{bar.v} + e_{h.wp}) + V_h \cdot e_{h.wp} - P_h \cdot (L_h + e_{v.wp} - y_{bar.left})}{e_{v.wp}}$$

$$V_v = \frac{187kip \cdot (17.6in + 7.1in) + 246kip \cdot 7.1in - 125kip \cdot (28.4in + 7.8in - 9.9in)}{7.8in} = 399kip$$

$$C_{RCC} = \sqrt{(V_h + P_v)^2 + (V_v + P_h)^2} = \sqrt{(246kip + 187kip)^2 + (399kip + 125kip)^2} = 680kip$$

G6.2.4a3 - Trial Force Substitution Cont.:



RCC Resultants Nonparallel Capacity (per plate)

Total member capacity

$2 \cdot 680\text{kip} = 1360\text{kip}$

Figure 17: Refined Corner Check Resultants with Resultants Not Parallel to Member

If the remaining stress checks are adequate, this combination of forces would provide a calculated capacity such that Horizontal Shear will control the load rating.

G6.2.4b Vertical Surface Check: Nonparallel Resultants

Constrain moments about work point to balance

$$\sigma_v = \frac{P_v}{A_v} = \frac{187\text{kip}}{16.4\text{in}^2} = 11.4\text{ksi} \qquad \tau_v = \frac{V_v}{A_v} = \frac{399\text{kip}}{16.4\text{in}^2} = 24.4\text{ksi}$$

$$\sigma_{vm.v} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{(11.4\text{ksi})^2 + 3\cdot(24.4\text{ksi})^2} = 43.8\text{ksi} \qquad \leq \; F_y = 53\,\text{ksi}$$

G6.2.4c Remaining Portion (Stub) Check - Nonparallel Resultants

Calculate equivalent concurrent forces for vertical and tension diagonal

$$F_{RCC.M3} = OpForce_{M3} \cdot \left| \frac{C_{RCC}}{OpForce_{M2}} \right| = 893kip \cdot \left| \frac{680kip}{-1050kip} \right| = 578kip$$

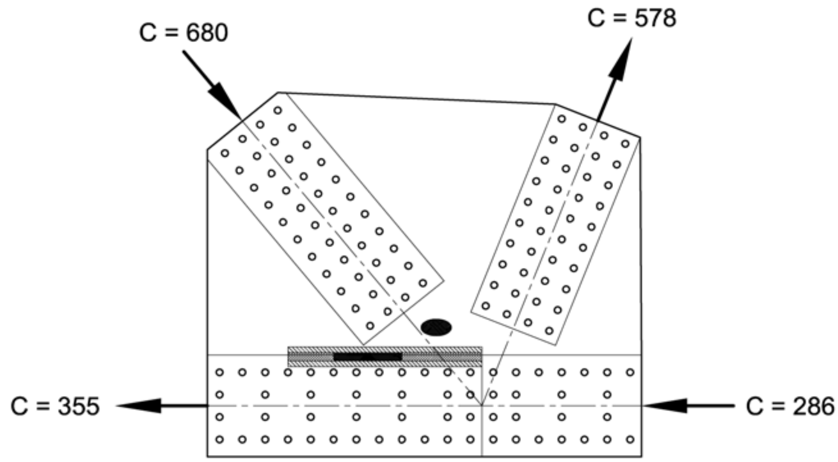


Figure 18: Concurrent Member Capacities (per plate) Based on Refined Corner Check (Subject to Stub Check and Buckling Check)

Check remaining portion of the gusset plate outside of the corner and chord. Select a Section Q that encompasses all forces applied by member M3.

$L_Q = 28.74$ in

$L_{SD} = 7.75$ in

$\theta_{M3} = 68.23 \cdot deg$

$L_{M3\_CC} = 10.6in$

$L_v = 34.0$ in

$e_{h.wp} = 7.1$ in

$e_{Q.wp} = e_{h.wp}$

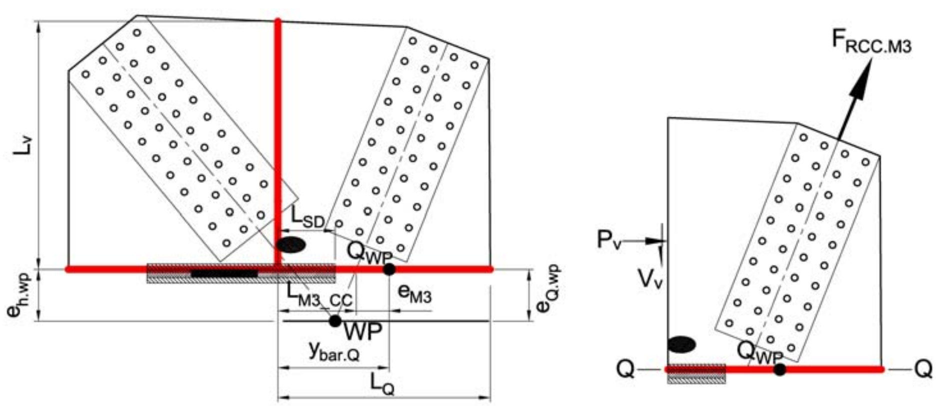


Figure 19: Remaining Gusset Plate Stub

Check the remaining portion of the gusset plate outside of the corner and chord.

$L_Q = 28.7$ in

$e_{M3} = 4.5$ in

Calculate forces P_Q and V_Q along Section Q

$$P_Q = F_{RCC.M3} \cdot \sin(\theta_{M3}) - V_v = 578kip \cdot \sin(68.2deg) - 399kip = 137kip$$

$$V_Q = F_{RCC.M3} \cdot \cos(\theta_{M3}) + P_v = 578kip \cdot \cos(68.2deg) + 187kip = 401kip$$

G6.2.4c Remaining Portion (Stub) Check- Nonparallel Resultants Cont.:

Calculate moment M_Q about Section Q

$$M_Q = P_v \cdot (y_{bar.v}) - V_v \cdot y_{bar.Q} + F_{RCC.M3} \cdot \sin(\theta_{M3}) \cdot e_{M3}$$

$$M_Q = 187\text{kip} \cdot (17.6\text{in}) - 399\text{kip} \cdot 15.1\text{in} + 578\text{kip} \cdot \sin(68.2\text{deg}) \cdot 4.5\text{in} = -330\text{kip} \cdot \text{in}$$

Determine section modulus and calculate bending and normal stresses

$$\sigma_P = \frac{P_Q}{A_Q} = \frac{137\text{kip}}{13.2\text{in}^2} = 10.4\text{ksi}$$

$$\sigma_M = \frac{|M_Q|}{S_Q} = \frac{|-330\text{kip} \cdot \text{in}|}{49.4\text{in}^3} = 6.7\text{ksi}$$

Since $\sigma_P + \sigma_M < F_y$ and $\sigma_M < \sigma_P$, use σ in von Mises equation based on 0.6*L (Refer to Appendix A)

$$\sigma_{0.6} = (\sigma_P - \sigma_M) + 0.6 \cdot [(\sigma_P + \sigma_M) - (\sigma_P - \sigma_M)]$$

$$\sigma_{0.6} = (10.4\text{ksi} - 6.7\text{ksi}) + 0.6 \cdot [(10.4\text{ksi} + 6.7\text{ksi}) - (10.4\text{ksi} - 6.7\text{ksi})] = 11.7\text{ksi}$$

$$\Omega = \sqrt{1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2} = \sqrt{1 - \left(\frac{11.7\text{ksi}}{53\text{ksi}}\right)^2} = 0.98$$

$$\tau_N = \Omega \cdot (0.58) \cdot F_y = 0.98 \cdot (0.58) \cdot 53\text{ksi} = 30.0\text{ksi}$$

Therefore, remaining portion of gusset plate is adequate for this combination of forces.

$$v_Q = \frac{V_Q}{A_Q} = \frac{401\text{kip}}{13.2\text{in}^2} = 30.4\text{ksi} \qquad \sim\le \qquad \tau_N = 30.0\,\text{ksi}$$

G6.2.4d Buckling Check: Nonparallel Resultants

Check buckling due to axial forces on surfaces (refer to Appendix B)

G6.2.3d1 Short Gap Buckling Check:

For this gusset plate, the short gap corresponds to the horizontal surface

$$F_{cr} = 38.0\text{ksi}$$ See Basic Corner Check

$$\sigma_h = \frac{P_h}{A_{eff}} = \frac{125\text{kip}}{12.3\text{in}^2} = 10.1\text{ksi} \qquad \tau_h = \frac{V_h}{A_{eff}} = \frac{245\text{kip}}{12.3\text{in}^2} = 19.9\text{ksi}$$

$$\sigma_{Princ} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{10.1\text{ksi}}{2} + \sqrt{\left(\frac{10.1\text{ksi}}{2}\right)^2 + (19.9\text{ksi})^2} = 25.6\text{ksi} \quad \le \quad F_{cr} = 38.0\text{ksi}$$

The principle stress is less than the critical buckling stress; therefore, buckling is not a concern.

G6.2.3d2 Long Gap Buckling Check

Treat as flat rectangular plate with one non-loaded edge fixed and the remaining edges clamped

Not a concern as $a/b \le 0.75$ See Basic Corner Check

Since buckling was not a concern for the Basic Corner Check, no reduction in calculated capacity is required.

$$C_{RCC} = 680\text{ kip}$$

RCC Resultants Nonparallel Capacity (per plate)

Total member capacity

$$2 \cdot 680\text{kip} = 1360\text{kip}$$

$$ORF_{RCC} = \frac{C_{RCC} - \gamma_{DL}\cdot\left|\frac{1}{2}DL_{M2}\right|}{\gamma_{LL}\cdot\left|\frac{1}{2}LL_{M2}\right|} = \frac{680\text{kip} - 1.3\cdot\left|\frac{1}{2}\cdot -646\text{kip}\right|}{1.3\cdot\left|\frac{1}{2}\cdot -162\text{kip}\right|} = 2.47$$

$$IRF_{RCC} = \frac{C_{RCC} - \gamma_{DL}\cdot\left|\frac{1}{2}DL_{M2}\right|}{\gamma_{InvLL}\cdot\left|\frac{1}{2}LL_{M2}\right|} = \frac{680\text{kip} - 1.3\cdot\left|\frac{1}{2}\cdot -646\text{kip}\right|}{2.17\cdot\left|\frac{1}{2}\cdot -162\text{kip}\right|} = 1.48$$

Because this result for the Refined Corner Check is greater than result from Horizontal Shear, no further iterations are necessary.

G6.2.5 Evaluation Summary:

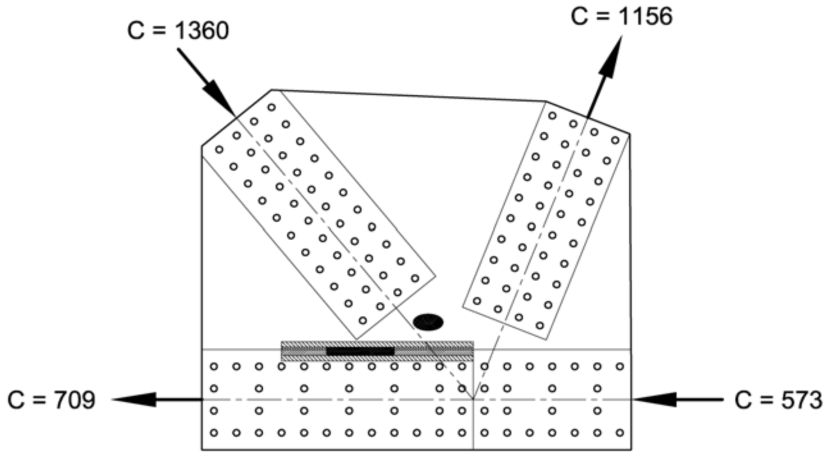


Figure 20: Concurrent Member Capacities Based on Refined Analysis (for Gusset Plate Pair)

Limit State	Gusset Plate Pair		
	Operating Rating	Inventory Rating	
Fasteners*	4.81	2.88	
Vertical Shear	3.60	2.16	Ignores splice plate
~~Horizontal Shear~~[1]	~~1.45~~	~~0.87~~	
~~Partial Shear Yield~~[2]	~~0.49~~	~~0.49~~	
~~Whitmore Compression~~[2]	~~1.04~~	~~0.62~~	
Tension*	6.76	4.05	
Block Shear*	3.99	2.39	
Chord Splice	60.0	36.0	
Horizontal Shear (Ω Calc.)	2.25	1.35	**Controls**
~~Basic Corner Check~~[3]	~~1.59~~	~~0.95~~	
Refined Corner Check	2.47	1.48	

[1] Superceded by Horizontal Shear with Ω calculated.
[2] Superceded by Basic Corner Check (see [3]).
[3] Superceded by final iteration of Refined Corner Check.
* Not affected by shown deterioration

By refining the analysis calculations using the approach presented above, a substantial increase in the Operating Rating can be achieved when considering the effects of deterioration.

Example 7 - Compact End Node Gusset Plate

Load Factor Rating (LFR) Method

Example 7 is a three member gusset plate at a bearing location (end node) with a short buckling length between members. It is a compact gusset plate with the diagonal member chamfered. Calculations apply to one of two gusset plates.

G7.1 Gusset Plate Material, Geometric, and Loading Properties:

Material Properties

$F_y = 33\text{ksi}$

$F_u = 60\text{ksi}$

$E = 29000\text{ksi}$

$\nu = 0.3$

Plate Thickness

$t = \frac{5}{8}\text{in}$

Member Angles

$\theta_{M2} = 45.45\text{deg}$

$\theta_{M3} = 87.17\text{deg}$

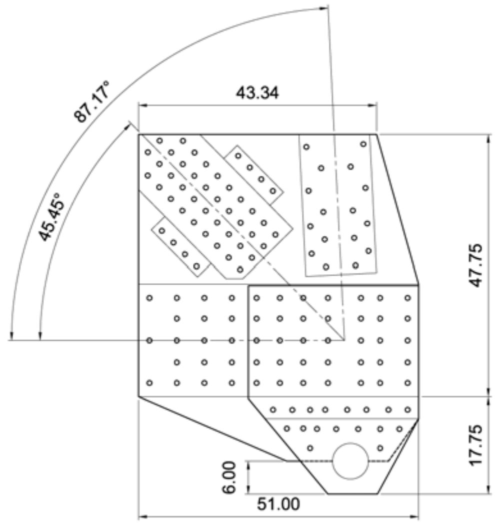


Figure 1: Basic Geometry of Gusset Plate

Note that a 3/4" doubler plate is shown at pin

Unfactored Member Forces Per Gusset Plate Pair

$LL_{M1} = 135\,\text{kip}$

$DL_{M1} = 278\,\text{kip}$

$LL_{M2} = -220\,\text{kip}$

$DL_{M2} = -406\,\text{kip}$

$LL_{M3} = -109\,\text{kip}$

$DL_{M3} = -39\,\text{kip}$

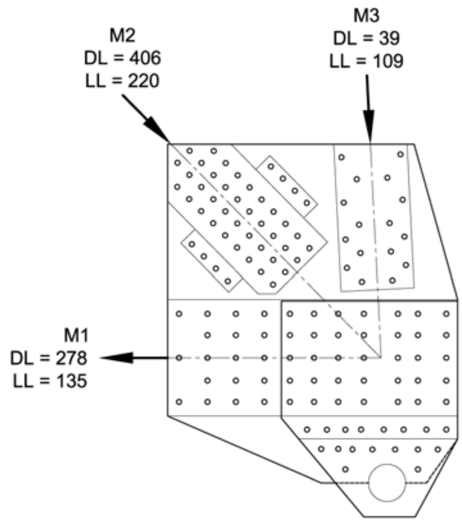


Figure 2: Concurrent Member Forces Transferred to Two Gusset Plates

Factored Forces Acting on Gusset Plate Pair

$InvForce_{M1} = \gamma_{InvLL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 2.17 \cdot 135kip + 1.3 \cdot 278kip$ $\qquad InvForce_{M1} = 654\,kip$

$OpForce_{M1} = \gamma_{LL} \cdot LL_{M1} + \gamma_{DL} \cdot DL_{M1} = 1.3 \cdot 135kip + 1.3 \cdot 278kip$ $\qquad OpForce_{M1} = 536\,kip$

$InvForce_{M2} = -1006\,kip$

$OpForce_{M2} = -814\,kip$

$InvForce_{M3} = -287\,kip$

$OpForce_{M3} = -192\,kip$

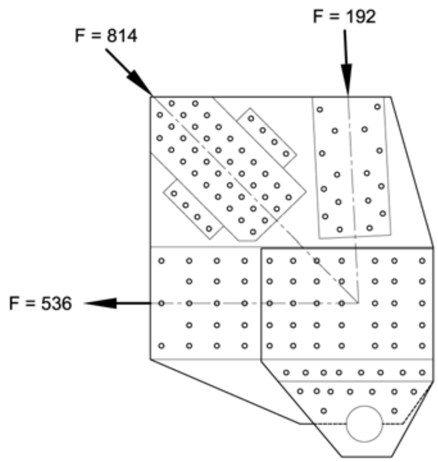


Figure 3: Concurrent Member Operating Forces Transferred to Two Gusset Plates

G7.2 Evaluation Approach:

In accordance with the *2014 Interim Revisions to the Manual for Bridge Evaluation, Second Edition,* the following gusset plate limit state checks were done:
(a) Fastener strength (L6B.2.6.1)
(b) Vertical shear resistance (L6B.2.6.3)
(c) Horizontal shear resistance (L6B.2.6.3)
(d) Partial shear yield resistance (L6B.2.6.3)
(e) Compressive (Whitmore) resistance (L6B.2.6.4)
(f) Tension strength (L6B.2.6.5) - Not Applicable
(g) Bock shear resistance (L6B.2.6.5)
(h) Chord splice capacity (L6B.2.6.6) - Not Applicable

Load Factor Rating Summary for Example 7

Limit State	**Gusset Plate Pair**		
	Operating Rating	**Inventory Rating**	
Fasteners	2.67	1.60	7/8 in. diam rivets
Vertical Shear	7.25	4.34	
Horizontal Shear	3.31	1.98	
Partial Shear Yield	1.09	0.65	**Controls**
Whitmore Compression	2.53	1.52	
Tension	-	-	
Block Shear	11.02	6.60	
Chord Splice	-		

When the Partial Shear Plane Yield and/or Whitmore Compression capacity checks control and indicate a less than acceptable rating, more rigorous evaluation should be performed. When evaluating a gusset plate at an end node such as is presented in this example, a more rigorous Horizontal Shear capacity should be determined.

The following more rigorous rating checks are performed in Example 7:
(1) Horizontal shear capacity - Ω calculated: Ω calculated: Supercedes Horizontal Shear with Ω = 0.88.
(2) Basic Corner Check capacity (BCC): Replaces Partial Shear Plane Yield and Whitmore Compression capacity che

G7.2.1 Horizontal Shear (AASHTO L6B.2.6.3 with Calculated Ω):

Global shear check along horizontal planes that are parallel with bottom chord. To determine the appropriate shear reduction factor, both moment and axial force acting on shear plane must be considered. An effective Ω will be calculated through an iterative approach for both the determination of the Operating Rating and Inventory Rating.

Because of the iterative approach, this check may be performed based on knowing the controlling force from other failure mechanism and using such a force as a starting force. The example below will not take advantage of such knowledge.

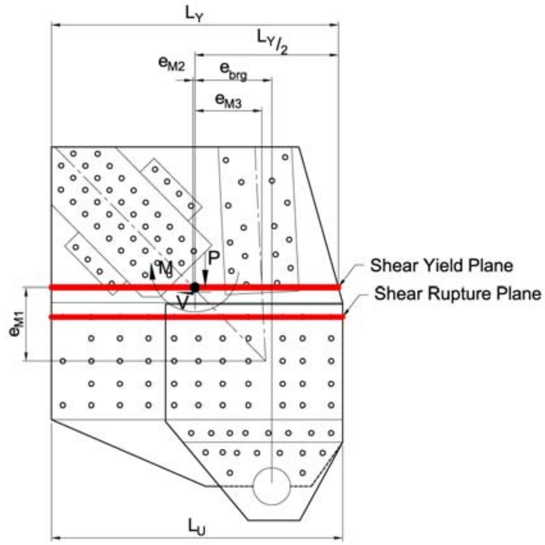


Figure 4: Horizontal Shear Between Web and Chord Members

G7.2.1a Horizontal Shear - Geometric Properties:

Account for the bottom chord not being horizontal

$\theta_{PanelPoint} = 2.83 \cdot deg$

Determine geometric properties of member forces relative to horizontal shear yield plane. Dimensions are positive if they are to the right of or above the plane's midpoint.

$L_Y = 50.3\,in$

$e_{M1} = -12.9\,in$

$e_{M2} = -0.3\,in$

$e_{M3} = 11.7\,in$

$e_{brg} = 13.5\,in$

Determine section properties

$A_g = t \cdot L_Y = \frac{5}{8} in \cdot 50.3 in = 31.4 in^2$

$S_g = \frac{{L_Y}^2 \cdot t}{6} = 263 in^3$

G7.2.1a Horizontal Shear - Geometric Properties Cont.:

$L_U = 51\,in$

$e_{M1.U} = -7.8\,in$

$e_{M2.U} = 4.4\,in$

$e_{M3.U} = 11.6\,in$

$e_{brg.U} = 13.1\,in$

$n_{hole} = 11$

$d_h = 1\,in$

$$A_n = t \cdot (L - n_{hole} \cdot d_h) = \frac{5}{8} in \cdot [51.0in - (11) \cdot 1.0in] = 25.0in^2$$

Determine section modulus - Note that distances reported in brackets in the numerator are the distance from the right edge of the gusset plate to the center of a particular hole.

$$y_{bar.rightt} = \frac{(2.0in + 7.0in + 10.5in + 16.5in + 21.0in + 25.25in + 29.5in + 34.0in + 39.0in + 44.0in + 49.0in) \cdot t \cdot d_h}{n_{hole} \cdot t \cdot d_h}$$

$y_{bar.right} = 25.25\,in$

$$I_r = \frac{1}{12} \cdot t \cdot L_U{}^3 - n_{hole} \cdot \frac{1}{12} \cdot t \cdot d_h{}^3 - \sum A_i (d_i)^2$$

$$I_n = \frac{1}{12} \cdot t \cdot L_U{}^3 - n_{hole} \cdot \frac{1}{12} \cdot t \cdot d_h{}^3 \; ... \\ + \left[t \cdot d_h \cdot \left[\begin{array}{l} (y_{bar.right} - 2.0in)^2 + (y_{bar.right} - 7.0in)^2 + (y_{bar.right} - 10.5in)^2 + (y_{bar.right} - 16.5in)^2 \; ... \\ + (y_{bar.right} - 21.0in)^2 + (y_{bar.right} - 25.25in)^2 + (y_{bar.right} - 29.5in)^2 + (y_{bar.right} - 34.0in)^2 \; ... \\ + (y_{bar.right} - 39.0in)^2 + (y_{bar.right} - 44.0in)^2 + (y_{bar.right} - 49.0in)^2 \end{array} \right] \right]$$

$$I_n = \frac{1}{12} \cdot \frac{5}{8} in \cdot (51.0in)^3 - 11 \cdot \frac{1}{12} \cdot \frac{5}{8} in \cdot (1in)^3 \; ... \\ + \left[\frac{5}{8} in \cdot 1in \cdot \left[\begin{array}{l} (25.25in - 2.0in)^2 + (25.25in - 7.0in)^2 + (25.25in - 10.5in)^2 + (25.25in - 16.5in)^2 \; ... \\ + (25.25in - 21.0in)^2 + (25.25in - 25.25in)^2 + (25.25in - 29.5in)^2 + (25.25in - 34.0in)^2 \; ... \\ + (25.25in - 39.0in)^2 + (25.25in - 44.0in)^2 + (25.25in - 49.0in)^2 \end{array} \right] \right]$$

$I_n = 5420in^4$

$$c_n = \max(y_{bar.right}, L_U - y_{bar.right}) = 25.75\,in$$

$$S_n = \frac{I_n}{c_n} = \frac{5420in^4}{25.75\,in} = 210in^3$$

G7.2.1b Horizontal Shear:

G7.2.1b1 Horizontal Shear Yield:

G7.2.1b1a Horizontal Shear Yield - First Iteration:

As a starting point, set the shear yield member forces equal to the shear yield member forces determined by following AASHTO L6B.2.6.3.

$$\phi_{yy} = 1.0$$

$$C_Y = \phi_{yy}\cdot(0.58)\cdot F_y\cdot A_g\cdot\Omega = 1.0\cdot(0.58)\cdot 33ksi\cdot 31.4in^2\cdot 0.88 = 529kip$$

Set shear from members M2 and M3 to shear yield capacity

$$C_Y = F_{M2}\cdot\cos(\theta_{M2}) + F_{M3}\cdot\cos(\theta_{M3})$$

Detemine force in member M3 in terms of the force in member M2 (note that the dead load is constant and only the live load scales in order to have the same rating value for all members).

$$F_{M3} = 1.3DL_{M3} + LL_{M3}\cdot\left(\frac{|F_{M2}| - 1.3\,|DL_{M2}|}{|LL_{M2}|}\right)$$

Substitute relationship and solve for F_{M2}

$$C_Y = F_{M2}\cdot\cos(\theta_{M2}) + \left[1.3DL_{M3} + LL_{M3}\cdot\left(\frac{|F_{M2}| - 1.3\,|DL_{M2}|}{|LL_{M2}|}\right)\right]\cdot\cos(\theta_{M3})$$

$$F_{M2} = -\frac{C_Y - \cos(\theta_{M3})\cdot\left(1.3\cdot|DL_{M3}| - \frac{1.3\cdot|DL_{M2}|\cdot|LL_{M3}|}{|LL_{M2}|}\right)}{\cos(\theta_{M2}) + \frac{|LL_{M3}|\cdot\cos(\theta_{M3})}{|LL_{M2}|}}$$

$$F_{M2} = -\frac{529kip - \cos(87.17deg)\cdot\left(1.3\cdot|-39kip| - \frac{1.3\cdot|-406kip|\cdot|-109kip|}{|-220kip|}\right)}{\cos(45.45deg) + \frac{|-109kip|\cdot\cos(87.17deg)}{|-220kip|}} = -743kip$$

Determine concurrent member forces and bearing reaction for this scaled load

$$F_{M1} = 1.3DL_{M1} + LL_{M1}\cdot\left(\frac{|F_{M2}| - 1.3\,|DL_{M2}|}{|LL_{M2}|}\right) = 1.3\cdot 278kip + 135kip\cdot\left(\frac{|-743kip| - 1.3\,|-406kip|}{|-220kip|}\right) = 493kip$$

$$F_{M3} = 1.3DL_{M3} + LL_{M3}\cdot\left(\frac{|F_{M2}| - 1.3\,|DL_{M2}|}{|LL_{M2}|}\right) = 1.3\cdot 39kip + 109kip\cdot\left(\frac{|-743kip| - 1.3\,|-406kip|}{|-220kip|}\right) = -157kip$$

$$R_{brg} = -\left(F_{M2}\cdot\sin(\theta_{M2} + \theta_{PanelPoint}) + F_{M3}\cdot\sin(\theta_{M3} + \theta_{PanelPoint})\right)$$

$$R_{brg} = -(-743kip\cdot\sin(45.45deg + 2.83deg) + -157kip\cdot\sin(87.17deg + 2.83deg)) = 712kip$$

G7.2.1b1a Horizontal Shear Yield - First Iteration Cont.:

Calculate the forces acting on the the shear yield plane

$$P_{Plane} = F_{M2} \cdot \sin(\theta_{M2}) + F_{M3} \cdot \sin(\theta_{M3}) = -743kip \cdot \sin(45.45deg) + -157kip \cdot \sin(87.17deg) = -686kip$$

$$V_{Plane} = \left|F_{M2} \cdot \cos(\theta_{M2}) + F_{M3} \cdot \cos(\theta_{M3})\right| = \left|-743kip \cdot \cos(45.45deg) + -157kip \cdot \cos(87.17deg)\right| = 529kip$$

$$M_{Plane} = F_{M2} \cdot \sin(\theta_{M2}) \cdot e_{M2} + F_{M3} \cdot \sin(\theta_{M3}) \cdot e_{M3} + R_{brg} \cdot \cos(\theta_{PanelPoint}) \cdot e_{brg} + F_{M1} \cdot e_{M1}$$

$$M_{Plane} = \begin{bmatrix} -743kip \cdot \sin(45.45deg)(-0.3in) + -157kip \cdot \sin(87.17deg) \cdot (11.7in) \ ... \\ + 712kip \cdot \cos(2.83deg) \cdot 13.5in + 493kip \cdot (-12.9in) \end{bmatrix} = 1550kip \cdot in$$

Calculate bending and normal stresses on shear plane

$$\sigma_P = \frac{\left|P_{Plane}\right|}{A_g} = \frac{\left|-686kip\right|}{31.4in^2} = 21.8ksi$$

$$\sigma_M = \frac{\left|M_{Plane}\right|}{S_g} = \frac{\left|1550kip \cdot in\right|}{263in^3} = 5.9ksi$$

Since $\sigma_P + \sigma_M < F_y$ and $\sigma_M < \sigma_P$, use σ in von Mises equation based on 0.6*L (Refer to Appendix A)

$$\sigma_{0.6} = (\sigma_P - \sigma_M) + 0.6 \cdot \left[(\sigma_P + \sigma_M) - (\sigma_P - \sigma_M)\right]$$

$$\sigma_{0.6} = (21.8ksi - 5.9ksi) + 0.6 \cdot [(21.8ksi + 5.9ksi) - (21.8ksi - 5.9ksi)] = 23.0ksi$$

$$\Omega = \sqrt{1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2} = \sqrt{1 - \left(\frac{23.0ksi}{33ksi}\right)^2} = 0.72$$

$$\tau_N = \Omega \cdot (0.58) \cdot F_y = 0.72 \cdot (0.58) \cdot 33ksi = 13.7ksi$$

Check shear on Section Q to see if it is less than 15.9 ksi

$$v_{Plane} = \frac{V_{Plane}}{A_g} = \frac{529kip}{31.4in^2} = 16.8ksi \qquad \geq \qquad \tau_N = 13.7\,ksi$$

Therefore, shear plane is overstressed for this combination of forces and the capacity must be recalculated.

G7.2.1b1b Horizontal Shear Yield - Second Iteration:

Determine the ratio of the decrease in shear force and reduce meber forces based on von Mises relationship as a second starting point

$$\text{Ratio} = \frac{\tau_N}{v_{Plane}} = \frac{13.7\text{ksi}}{16.8\text{ksi}} = 0.81$$

$$F_{M2} = -743\text{kip}\cdot\left[1 + \frac{(0.81 - 1)}{\sqrt{3}}\right] = -663\text{kip}$$

Determine concurrent member forces and bearing reaction for this scaled load

$$F_{M1} = 1.3DL_{M1} + LL_{M1}\cdot\left(\frac{|F_{M2}| - 1.3\,|DL_{M2}|}{|LL_{M2}|}\right) = 1.3\cdot 278\text{kip} + 135\text{kip}\cdot\left(\frac{|-663\text{kip}| - 1.3\,|-406\text{kip}|}{|-220\text{kip}|}\right) = 444\text{kip}$$

$$F_{M3} = 1.3DL_{M3} + LL_{M3}\cdot\left(\frac{|F_{M2}| - 1.3\,|DL_{M2}|}{|LL_{M2}|}\right) = 1.3\cdot 39\text{kip} + 109\text{kip}\cdot\left(\frac{|-663\text{kip}| - 1.3\,|-406\text{kip}|}{|-220\text{kip}|}\right) = -117\text{kip}$$

$$R_{brg} = -\left(F_{M2}\cdot\sin\left(\theta_{M2} + \theta_{PanelPoint}\right) + F_{M3}\cdot\sin\left(\theta_{M3} + \theta_{PanelPoint}\right)\right)$$

$$R_{brg} = -(-663\text{kip}\cdot\sin(45.45\text{deg} + 2.83\text{deg}) + -117\text{kip}\cdot\sin(87.17\text{deg} + 2.83\text{deg})) = 613\text{kip}$$

Calculate the forces acting on the the shear yield plane

$$P_{Plane} = F_{M2}\cdot\sin\left(\theta_{M2}\right) + F_{M3}\cdot\sin\left(\theta_{M3}\right) = -663\text{kip}\cdot\sin(45.45\text{deg}) + -117\text{kip}\cdot\sin(87.17\text{deg}) = -590\text{kip}$$

$$V_{Plane} = \left|F_{M2}\cdot\cos\left(\theta_{M2}\right) + F_{M3}\cdot\cos\left(\theta_{M3}\right)\right| = |-663\text{kip}\cdot\cos(45.45\text{deg}) + -117\text{kip}\cdot\cos(87.17\text{deg})| = 471\text{kip}$$

$$M_{Plane} = F_{M2}\cdot\sin\left(\theta_{M2}\right)\cdot e_{M2} + F_{M3}\cdot\sin\left(\theta_{M3}\right)\cdot e_{M3} + R_{brg}\cdot\cos\left(\theta_{PanelPoint}\right)\cdot e_{brg} + F_{M1}\cdot e_{M1}$$

$$M_{Plane} = \begin{bmatrix} -663\text{kip}\cdot\sin(45.45\text{deg})(-0.3\text{in}) + -117\text{kip}\cdot\sin(87.17\text{deg})\cdot(11.7\text{in}) \;... \\ + 613\text{kip}\cdot\cos(2.83\text{deg})\cdot 13.5\text{in} + 444\text{kip}\cdot(-12.9\text{in}) \end{bmatrix} = 1290\text{kip}\cdot\text{in}$$

Calculate bending and normal stresses on shear plane

$$\sigma_P = \frac{|P_{Plane}|}{A_g} = \frac{|-590\text{kip}|}{31.4\text{in}^2} = 18.8\text{ksi}$$

$$\sigma_M = \frac{|M_{Plane}|}{S_g} = \frac{|1290\text{kip}\cdot\text{in}|}{263\text{in}^3} = 4.9\text{ksi}$$

G7.2.1b1b Horizontal Shear Yield - Second Iteration Cont.:

Since $\sigma_P + \sigma_M < F_y$ and $\sigma_M < \sigma_P$, use σ in von Mises equation based on 0.6*L (Refer to Appendix A)

$$\sigma_{0.6} = (\sigma_P - \sigma_M) + 0.6 \cdot [(\sigma_P + \sigma_M) - (\sigma_P - \sigma_M)]$$

$$\sigma_{0.6} = (18.8\text{ksi} - 4.9\text{ksi}) + 0.6 \cdot [(18.8\text{ksi} + 4.9\text{ksi}) - (18.8\text{ksi} - 4.9\text{ksi})] = 19.8\text{ksi}$$

$$\Omega = \sqrt{1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2} = \sqrt{1 - \left(\frac{19.8\text{ksi}}{33\text{ksi}}\right)^2} = 0.80$$

$$\tau_N = \Omega \cdot (0.58) \cdot F_y = 0.80 \cdot (0.58) \cdot 33\text{ksi} = 15.3\text{ksi}$$

Check shear on Section Q to see if it is less than 14.9 ksi

$$v_{Plane} = \frac{V_{Plane}}{A_g} = \frac{471\text{kip}}{31.4\text{in}^2} = 15.0\text{ksi} \qquad \leq \qquad \tau_N = 15.3\text{ ksi}$$

Therefore, shear plane is not overstressed for this combination of forces and is relatively close to the final answer ($F_{M2} = 672$ kip).

Determine shear capacity of plane

$$\phi_{vy} = 1.0$$

$$C_Y = \phi_{vy} V_{Plane} = 1.0 \cdot 471\text{kip} = 471\text{kip}$$

G7.2.1b2 Horizontal Shear Rupture:

G7.2.1b2a Horizontal Shear Rupture - First Iteration:

As a starting point, set the shear rupture member forces equal to the shear yield member forces while accounting for ϕ_{vu}.

$$\phi_{vu} = 0.85$$

$$F_{M2.U} = \frac{F_{M2}}{\phi_{vu}} = \frac{-663\text{kip}}{0.85} = -780\text{kip}$$

Determine concurrent member forces and bearing reaction for this scaled load (these will match the shear yield forces in this iteration)

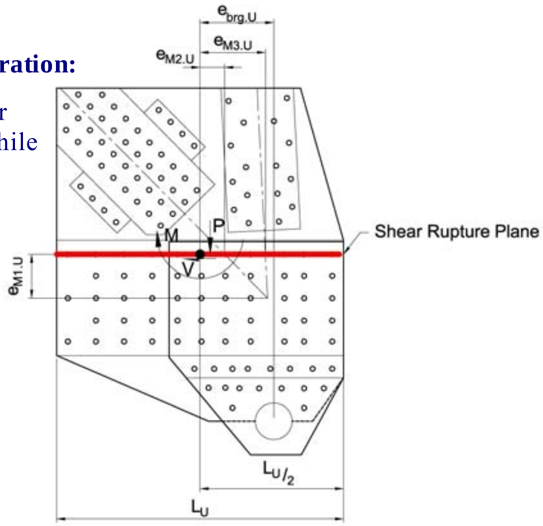


Figure 5: Horizontal Shear Rupture Between Web and Chord Members

$$F_{M1.U} = 1.3DL_{M1} + LL_{M1}\cdot\left(\frac{\left|F_{M2.U}\right| - 1.3\left|DL_{M2}\right|}{\left|LL_{M2}\right|}\right) = 1.3\cdot 278\text{kip} + 135\text{kip}\cdot\left(\frac{\left|-780\text{kip}\right| - 1.3\left|-406\text{kip}\right|}{\left|-220\text{kip}\right|}\right) = 515\text{ki}$$

$$F_{M3.U} = 1.3DL_{M3} + LL_{M3}\cdot\left(\frac{\left|F_{M2.U}\right| - 1.3\left|DL_{M2}\right|}{\left|LL_{M2}\right|}\right) = 1.3\cdot 39\text{kip} + 109\text{kip}\cdot\left(\frac{\left|-780\text{kip}\right| - 1.3\left|-406\text{kip}\right|}{\left|-220\text{kip}\right|}\right) = -175\text{ki}$$

$$R_{brg.U} = -\left(F_{M2.U}\cdot\sin\left(\theta_{M2} + \theta_{PanelPoint}\right) + F_{M3.U}\cdot\sin\left(\theta_{M3} + \theta_{PanelPoint}\right)\right)$$

$$R_{brg.U} = -(-780\text{kip}\cdot\sin(45.45\text{deg} + 2.83\text{deg}) + -175\text{kip}\cdot\sin(87.17\text{deg} + 2.83\text{deg})) = 758\text{kip}$$

Calculate the forces acting on the the shear yield plane

$$P_{Plane.U} = F_{M2.U}\cdot\sin\left(\theta_{M2}\right) + F_{M3.U}\cdot\sin\left(\theta_{M3}\right) = -780\text{kip}\cdot\sin(45.45\text{deg}) + -175\text{kip}\cdot\sin(87.17\text{deg}) = -731\text{kip}$$

$$V_{Plane.U} = \left|F_{M2.U}\cdot\cos\left(\theta_{M2}\right) + F_{M3.U}\cdot\cos\left(\theta_{M3}\right)\right| = \left|-780\text{kip}\cdot\cos(45.45\text{deg}) + -175\text{kip}\cdot\cos(87.17\text{deg})\right| = 556\text{kip}$$

$$M_{Plane.U} = F_{M2.U}\cdot\sin\left(\theta_{M2}\right)\cdot e_{M2.U} + F_{M3.U}\cdot\sin\left(\theta_{M3}\right)\cdot e_{M3.U} + R_{brg.U}\cdot\cos\left(\theta_{PanelPoint}\right)\cdot e_{brg.U} + F_{M1}\cdot e_{M1.U}$$

$$M_{Plane.U} = \begin{bmatrix} -780\text{kip}\cdot\sin(45.45\text{deg})(-0.3\text{in}) + -175\text{kip}\cdot\sin(87.17\text{deg})\cdot(11.7\text{in}) \ldots \\ + 758\text{kip}\cdot\cos(2.83\text{deg})\cdot 13.5\text{in} + 515\text{kip}\cdot(-12.9\text{in}) \end{bmatrix} = 1440\text{kip}\cdot\text{in}$$

Calculate bending and normal stresses on shear plane

$$\sigma_P = \frac{\left|P_{Plane.U}\right|}{A_n} = \frac{\left|-731\text{kip}\right|}{25.0\text{in}^2} = 29.3\text{ksi}$$

$$\sigma_M = \frac{\left|M_{Plane.U}\right|}{S_n} = \frac{\left|1440\text{kip}\cdot\text{in}\right|}{210\text{in}^3} = 6.9\text{ksi}$$

G7.2.1b2a Horizontal Shear Rupture - First Iteration Cont.:

Since $\sigma_P + \sigma_M < F_y$ and $\sigma_M < \sigma_P$, use σ in von Mises equation based on 0.6*L (Refer to Appendix A)

$$\sigma_{0.6} = (\sigma_P - \sigma_M) + 0.6 \cdot \left[(\sigma_P + \sigma_M) - (\sigma_P - \sigma_M) \right]$$

$$\sigma_{0.6} = (29.3\text{ksi} - 6.9\text{ksi}) + 0.6 \cdot [(29.3\text{ksi} + 6.9\text{ksi}) - (29.3\text{ksi} - 6.9\text{ksi})] = 30.6\text{ksi}$$

$$\Omega = \sqrt{1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2} = \sqrt{1 - \left(\frac{30.6\text{ksi}}{60\text{ksi}}\right)^2} = 0.86$$

$$\tau_N = \Omega \cdot (0.58) \cdot F_u = 0.86 \cdot (0.58) \cdot 60\text{ksi} = 29.9\text{ksi}$$

Check shear on Section Q to see if it is less than 15.9 ksi

$$v_{Plane} = \frac{V_{Plane}}{A_n} = \frac{556\text{kip}}{25.0\text{in}^2} = 22.3\text{ksi} \qquad \le \qquad \tau_N = 29.9\text{ ksi}$$

Therefore, the shear plane is not overstressed for this combination of forces when considering shear rupture and because shear rupture does not control the capacity, there is no need to recalculate.

Determine shear capacity of plane

$$\phi_{vu} = 0.85$$

$$C_U = \phi_{vy} V_{Plane} = 0.85 \cdot 556\text{kip} = 473\text{kip}$$

$$C_{HS} = \min(C_Y, C_U) = \min(471\text{kip}, 473\text{kip}) = 471\text{kip}$$

Determine capacity of member M2 based on Horizontal Shear

Horizontal Shear Capacity (per plate)

$$C_{HS.M2} = |F_{M2}| = 663\text{kip}$$

Total member capacity

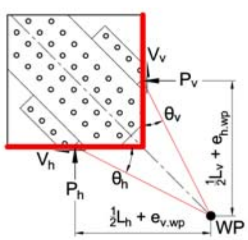
$$2 \cdot 663\text{kip} = 1327\text{kip}$$

$$ORF_{HS} = \frac{C_{HS.M2} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{663\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -406\text{kip}\right|}{1.3 \cdot \left|\frac{1}{2} \cdot -220\text{kip}\right|} = 2.79$$

$$IRF_{HS} = \frac{C_{HS.M2} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{663\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -406\text{kip}\right|}{2.17 \cdot \left|\frac{1}{2} \cdot -220\text{kip}\right|} = 1.67$$

G7.2.2 Basic Corner Check:

The Basic Corner Check is a first-principles analytical approach utilizing fundamental steel design theory to conservatively calculate gusset plate limit state capacities at critical cross sections. This check is used to evaluate equilibrium and stability of a gusset plate "corner" bounded by horizontal and vertical planes that create the smallest section encompassing all fasteners of the diagonal member. The diagonal member force is assumed to be resisted by a combination of shear and normal forces acting on the vertical and horizontal surfaces bounding the "corner". Von Mises stress calculated on the surfaces is limited to the yield strength of the gusset plate. For simplicity and to avoid bending in the members, the resultant of each surface must pass through the work point. The "corner" can be adjusted in terms of location and plate thickness to accommodate deterioration.



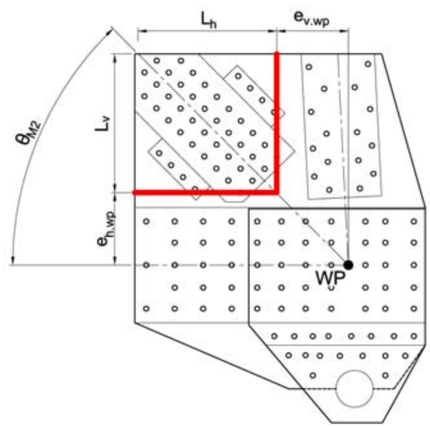


Figure 6: Basic Corner Check for Diagonal Member M2

Calculate resultant angles from the work point

$L_h = 24.9\,\text{in}$ $\qquad e_{h.wp} = 12.9\,\text{in}$

$L_v = 24.6\,\text{in}$ $\qquad e_{v.wp} = 12.6\,\text{in}$

$$\theta_h = \operatorname{atan}\left(\frac{e_{h.wp}}{\frac{L_h}{2} + e_{v.wp}}\right) = \operatorname{atan}\left(\frac{12.9\text{in}}{\frac{24.9\text{in}}{2} + 12.6\text{in}}\right) = 27.2\text{deg}$$

$$\theta_v = \operatorname{atan}\left(\frac{e_{v.wp}}{\frac{L_v}{2} + e_{h.wp}}\right) = \operatorname{atan}\left(\frac{12.6\text{in}}{\frac{24.6\text{in}}{2} + 12.9\text{in}}\right) = 26.5\text{deg}$$

G7.2.2a Vertical Surface Check:

Since $L_v < L_h$ set von Mises stress on vertical surface equal to plate yield strength. After stresses on both surfaces are determined; verify assumption that vertical surface is critical (i.e. reaches von Mises yield before horizontal surface).

$$P_v = V_v \cdot \tan(\theta_v)$$

Substitute P_v as a function of V_v and set the von Mises stress to yield

$$F_y = 33\text{ksi} = \sigma_{vm} = \sqrt{\sigma_v^2 + 3\tau_v^2} = \sqrt{\left(\frac{P_v}{L_v \cdot t}\right)^2 + 3\cdot\left(\frac{V_v}{L_v \cdot t}\right)^2} = \sqrt{\left(\frac{V_v \cdot \tan(\theta_v)}{L_v \cdot t}\right)^2 + 3\cdot\left(\frac{V_v}{L_v \cdot t}\right)^2}$$

Rearrange terms and solve for V_v

$$V_v = \frac{L_v \cdot F_y \cdot t}{\sqrt{\tan(\theta_v)^2 + 3}} = \frac{24.6\text{in} \cdot 33\text{ksi} \cdot \frac{5}{8}\text{in}}{\sqrt{\tan(26.5\text{deg})^2 + 3}} = 282\text{kip}$$

Solve for P_v

$$P_v = V_v \cdot \tan(\theta_v) = 282\text{kip} \cdot \tan(26.5\text{deg}) = 140\text{kip}$$

Calculate shear and normal stresses on vertical surface (to use when checking buckling strength)

$$\sigma_v = \frac{P_v}{L_v \cdot t} = \frac{140\text{kip}}{(24.6\text{in}) \cdot \left(\frac{5}{8}\text{in}\right)} = 9.1\text{ksi} \qquad \tau_v = \frac{V_v}{L_v \cdot t} = \frac{282\text{kip}}{(24.6\text{in}) \cdot \left(\frac{5}{8}\text{in}\right)} = 18.3\text{ksi}$$

G7.2.2b3 Determine Forces on Horizontal Surface:

Determine forces and stresses on horizontal surface based on vertical surface forces and stated constraints (i.e. force resultants to pass thru workpoint).

Check the horizontal surface:

$$P_h = V_h \cdot \tan(\theta_h)$$

Constrain final resultant to act along member and substitute P_h as a function of V_h

$$\theta_{M2} = \text{atan}\left(\frac{V_v + P_h}{P_v + V_h}\right) = \text{atan}\left(\frac{V_v + V_h \cdot \tan(\theta_h)}{P_v + V_h}\right) = \text{atan}\left(\frac{282\text{kip} + V_h \cdot \tan(27.2\text{deg})}{140\text{kip} + V_h}\right)$$

Rearrange terms and solve for V_h. Substitute values obtained from previously solving P_v and V_v.

$$V_h = \frac{V_v - P_v \cdot \tan(\theta_{M2})}{\tan(\theta_{M2}) - \tan(\theta_h)} = \frac{282\text{kip} - 140\text{kip} \cdot \tan(45.45\text{deg})}{\tan(45.45\text{deg}) - \tan(27.2\text{deg})} = 277\text{kip}$$

Solve for P_h

$$P_h = V_h \cdot \tan(\theta_h) = 277\text{kip} \cdot \tan(27.2\text{deg}) = 143\text{kip}$$

G7.2.2b3 Determine Forces on Horizontal Surface Cont.:

Calculate shear and normal stresses on vertical surface

$$\sigma_h = \frac{P_h}{L_h \cdot t} = \frac{143\text{kip}}{(24.9\text{in})\cdot\left(\frac{5}{8}\text{in}\right)} = 9.2\text{ksi} \qquad \tau_{hi} = \frac{V_h}{L_h \cdot t} = \frac{277\text{kip}}{(24.9\text{in})\cdot\left(\frac{5}{8}\text{in}\right)} = 17.8\text{ksi}$$

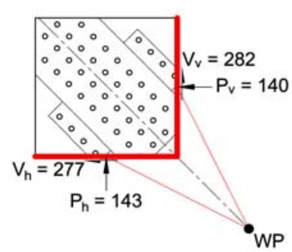
Calculate von Mises stress

$$\sigma_{vm.h} = \sqrt{\sigma_h^{\,2} + 3\tau_h^{\,2}} = \sqrt{(9.2\text{ksi})^2 + 3\cdot(17.8\text{ksi})^2} = 32.2\text{ksi} \qquad \le \qquad F_y = 33\,\text{ksi}$$

Since von Mises stress on horizontal surface is less than yield strength of the gusset plate, the vertical surface controls. If this had not been the case, the Von Mises stress calculated on the horizontal surface would have been greater than the yield stress. The previous process would have been modified by first setting the von Mises stress on the horizontal surface to the yield stress and then determining the necessary resultants on the vertical surface to balance the moment about the work point.

Substitute corresponding solved forces to determine member resultant force.

$$C_{BCCvM} = \sqrt{\left(V_h + P_v\right)^2 + \left(V_v + P_h\right)^2} = \sqrt{(277\text{kip} + 140\text{kip})^2 + (282\text{kip} + 143\text{kip})^2} = 595\text{kip}$$



BCC von Mises Capacity (per plate)

Total member capacity
$2\cdot595\text{kip} = 1191\text{kip}$

Figure 7: Basic Corner Check Resultants for Diagonal Member M2

G7.2.2c BCC Buckling Check:

Check plate buckling due to axial forces on Basic Corner Check surfaces (refer to Appendix B). If buckling controls, then von Mises stresses must be adjusted.

Evaluate buckling capacity of plate regions defined by the short and long span between the Basic Corner Check surfaces and adjacent member connections.

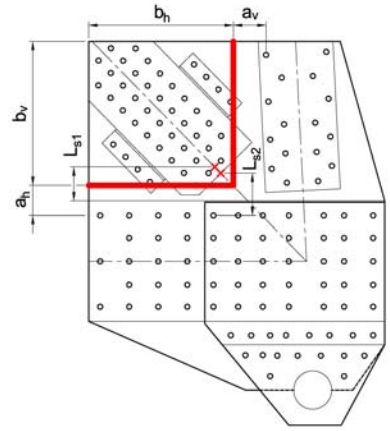


Figure 8: Corner Check Buckling Lengths

G7.2.2c1 Short Span Buckling Check:

For this gusset plate, the short span corresponds to the horizontal surface ($a_h < a_v$). a_h and a_v are defined as the distances from the respective Corner Check surface to the parallel line passing through the nearest fastener in an adjacent member.

$$L_s = \frac{L_{s1} + L_{s2}}{2} = \frac{7.2\text{in} + 6.1\text{in}}{2} = 6.6\text{in}$$

$$r = \frac{t}{\sqrt{12}} = \frac{\frac{5}{8}\text{in}}{\sqrt{12}} = 0.18\text{in}$$

Short span controls sidesway buckling, and rotation at each end is restrained. Therefore, K = 1.0 used.

$$F_e = \frac{\pi^2 \cdot E}{\left(\frac{K \cdot L_s}{r}\right)^2} = \frac{\pi^2 \cdot 29000\text{ksi}}{\left(\frac{1.0 \cdot 6.6\text{in}}{0.18\text{in}}\right)^2} = 212\text{ksi}$$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2 \cdot \sqrt{2}}\right) = 36.4\text{ksi} \cdot \left(1 - \frac{\sqrt{\frac{33\text{ksi}}{212\text{ksi}}}}{2 \cdot \sqrt{2}}\right) = 28.4\text{ksi}$$

$$\sigma = \sigma_h = 9.2\text{ksi}$$

$$\tau = \tau_h = 17.8\text{ksi}$$

$$\sigma_{Princ} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{9.2\text{ksi}}{2} + \sqrt{\left(\frac{9.2\text{ksi}}{2}\right)^2 + (17.8\text{ksi})^2} = 23.0\text{ksi} \qquad \le \qquad F_{cr} = 28.7\text{ksi}$$

Principle stress is less than the critical buckling stress; therefore, buckling of short span does not control.

G7.2.2c2 Long Span Buckling Check:

Treat long span as flat rectangular plate with one non-loaded edge fixed and the remaining edges clamped (dashed curve D in Figure 9)

Long Span Length (Figure 9)

$$a = a_v = 6.6\text{in}$$

Length of Long Side Surface (Figure 9)

$$b = L_v = 24.6\text{in}$$

$$\frac{a}{b} = 0.27$$

Because a/b is less than 0.75 (where k curve is nearly asymptotic), buckling of long span plate is not a concern. Otherwise calculate k as follows (using an approximate best fit function of dashed curve D in Figure 9):

$$k = 4.64 \cdot \left(\frac{a}{b}\right)^{-1.106}$$

$$F_e = \frac{k \cdot \pi^2 \cdot E}{12\left(1 - \nu^2\right) \cdot \left(\frac{b}{t}\right)^2}$$

$$F_{cr} = F_y \cdot \left(1 - \frac{\sqrt{\frac{F_y}{F_e}}}{2 \cdot \sqrt{2}}\right)$$

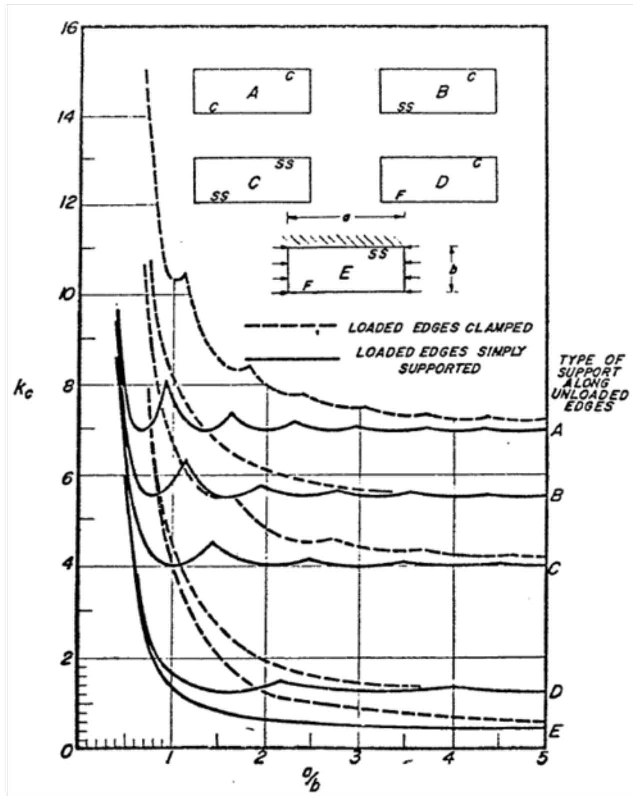


Figure 9: Elastic Buckling Coefficients [2]

Compare calculated principle stress to critical stress.

$$\sigma_{Princ} = \frac{\sigma_v}{2} + \sqrt{\left(\frac{\sigma_v}{2}\right)^2 + \tau_v^{\,2}} \quad \le \quad F_{cr}$$

G7.2.2 Basic Corner Check Cont.:

Since buckling of the short and long spans are not a concern for the Basic Corner Check, no reduction in calculated capacity is required and capacity calculated using von Mises stress applies.

$C_{BCC} = 595\,\text{kip}$

BCC Resultant Capacity (per plate)

Total member capacity

$2 \cdot 595\text{kip} = 1191\text{kip}$

$$ORF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{LL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{595\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -406\text{kip}\right|}{1.3 \cdot \left|\frac{1}{2} \cdot -220\text{kip}\right|} = 2.32$$

$$IRF_{BCC} = \frac{C_{BCC} - \gamma_{DL} \cdot \left|\frac{1}{2} DL_{M2}\right|}{\gamma_{InvLL} \cdot \left|\frac{1}{2} LL_{M2}\right|} = \frac{595\text{kip} - 1.3 \cdot \left|\frac{1}{2} \cdot -406\text{kip}\right|}{2.17 \cdot \left|\frac{1}{2} \cdot -220\text{kip}\right|} = 1.39$$

If an increased rating factor is required, perform a Refined Corner Check.

[2] George Gerard and Herbert Becker. *Handbook of Structural Stability*, Part I - Buckling of Flat Plates, Tech. Note 3871, National Advisory Committee for Aeronautics, Washington, D.C., July 1957.

G7.2.3 Evaluation Summary:

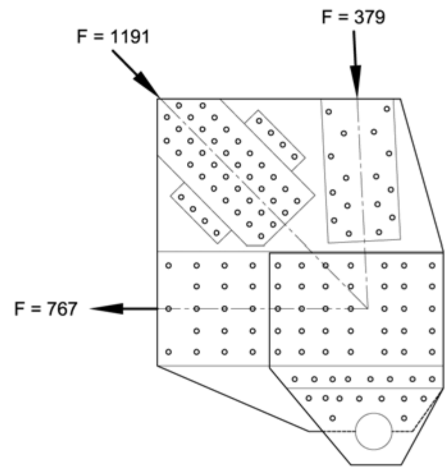


Figure 10: Concurrent Member Capacities Based on Refined Analysis (for Gusset Plate Pair)

Limit State	Gusset Plate Pair	
	Operating Rating	Inventory Rating
Fasteners	2.67	1.60
Vertical Shear	7.25	4.34
~~Horizontal Shear~~[1]	~~3.31~~	~~1.98~~
~~Partial Shear Yield~~[2]	~~1.09~~	~~0.65~~
~~Whitmore Compression~~[2]	~~2.53~~	~~1.52~~
Tension	-	-
Block Shear	11.02	6.60
Chord Splice	-	-
Horizontal Shear (Ω Calc.)	2.79	1.67
Basic Corner Check	2.32	1.39

Controls

[1] Superceded by Horizontal Shear with Ω calculated.
[2] Superceded by Basic Corner Check.

By refining the analysis calculations using the approach presented above, a greater than 100% increase in the Operating Rating can be achieved. This could further increase (if required) by performing a Refined Corner Check.

APPENDIX A

Accounting for the Interaction of Shear and Normal Forces on Various Surfaces

When evaluating the state of stress on a surface that carries both shear and moment, but no net normal force, the Guide uses the shear/moment interaction equation developed by Drucker (Drucker, 1956). When a surface is subjected to shear, moment and a net normal force, another evaluation method is needed.

When both moment and normal forces act on a surface, the resulting normal stress at any particular Point X along the surface (σ_X) can be determined. Then, using the von Mises interaction, the maximum sustainable shear stress at Point X (τ_X) can be calculated. If Point X represents the centroid of a segment of the surface with a length L_X, the shear strength of the segment (V_X) would be calculated as follows: $V_X = \tau_X(L_X)$. If this is done at a series of points covering the entire surface (e.g., at 10 points, each with a tributary length of one tenth of the surface length), the shear strength of the entire surface can be calculated as the sum of the segment shear strengths. In this way, the interaction of moment, normal force and shear can be accounted for on any surface.

The process described above is rather cumbersome to apply. Therefore, various combinations of moment and normal force were evaluated in order to develop formulas for calculating the shear strength under almost any practical set of conditions.

A	=	*area of surface (in.^{2})*
F_y	=	*yield stress (ksi) (ksi)*
L	=	*length of surface (in.)*
L_{seg}	=	*length of segment where $\sigma \leq F_y$ (in.)*
M	=	*moment demand (k-in.)*
M_P	=	*plastic moment (k-in.)*
P	=	*axial force (kip)*
S	=	*elastic section modulus (in^3)*
Z	=	*plastic section modulus (in^3)*
f_a	=	*axial stress (ksi)*
f_b	=	*bending stress (ksi)*
σ	=	*normal stress (ksi)*
$\sigma_{0.6}$	=	*normal stress at 0.6L or 0.6L_{seg} (ksi)*
σ_{MAX}	=	*maximum normal stress (ksi)*
σ_{MIN}	=	*minimum normal stress (ksi)*
τ	=	*shear stress (ksi)*
τ_p	=	*plastic shear stress (ksi)*

The von Mises interaction relates the maximum sustainable normal and shear stresses using the following formula:

$$\sigma^2 + 3\tau^2 = {F_y}^2$$

Rearranging terms gives available shear strength in terms of F_Y and normal stress.

$$\tau^2 = \frac{{F_y}^2 - \sigma^2}{3} = \frac{{F_y}^2}{3}\left[1 - \frac{\sigma^2}{{F_y}^2}\right]$$

> *Note: when normal stress is zero;* $\tau = \tau_P = \frac{F_y}{\sqrt{3}} = 0.58F_y$ *which is the formula used by most structural steel standards to relate shear strength to tensile strength*

Simplifying, a ratio of the shear stress to plastic shear stress is obtained.

$$\frac{\tau^2}{{\tau_p}^2} = 1 - \frac{\sigma^2}{{F_y}^2}$$

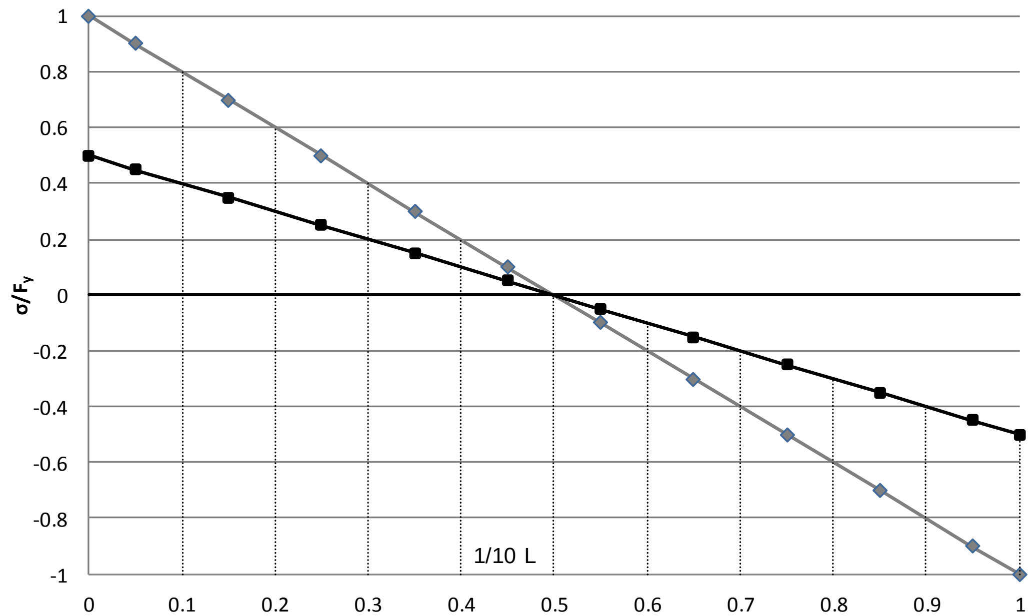
$$\frac{\tau}{\tau_p} = \left[1 - \frac{\sigma^2}{{F_y}^2}\right]^{\frac{1}{2}}$$

This equation is similar to Drucker's interaction.

$$\frac{\tau}{\tau_p} = \left[1 - \frac{M}{M_P}\right]^{\frac{1}{4}}$$

Various combinations of normal force and bending moment are evaluated below.

Case 1: No net normal force, $\boldsymbol{\sigma_{MAX} \leq F_y}$



Ω_{vm} (σ_{Max}= F_y) =	0.44	0.71	0.87	0.95	0.99	0.99	0.95	0.87	0.71	0.44
Ω_{vm} (σ_{Max}= 0.50F_y) =	0.89	0.94	0.97	0.99	1.00	1.00	0.99	0.97	0.94	0.89

Figure A1. Case 1 von Mises shear strengths for each segment

Table A1. Net τ/τ_P

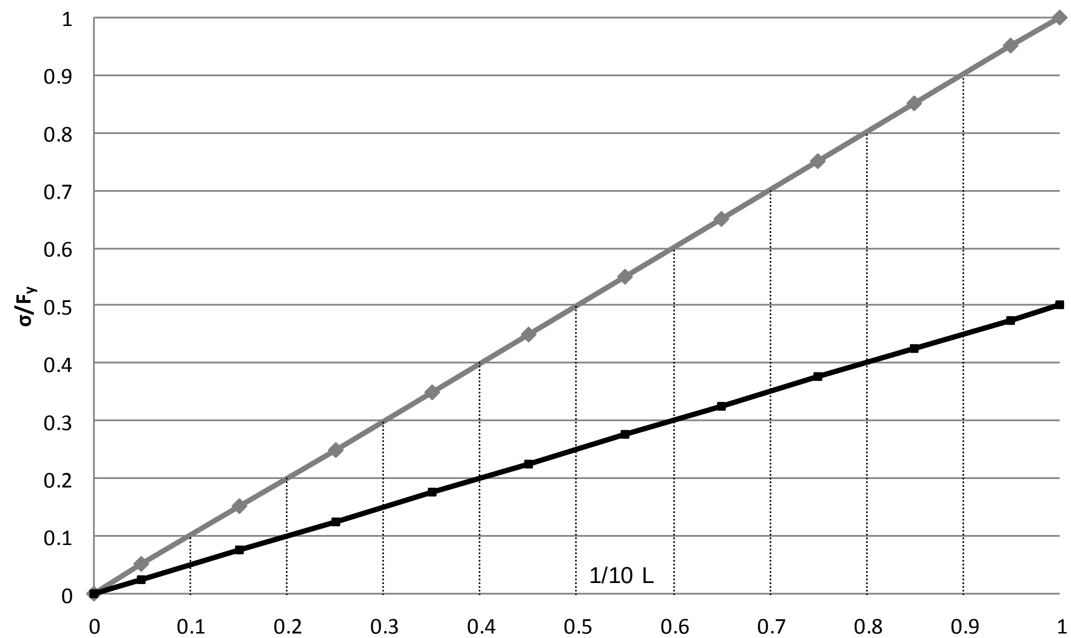
Case	von Mises	Drucker
σ_{Max}= F_y	0.79	0.76
σ_{Max}= 0.5F_y	0.96	0.90

For Drucker: $\frac{\tau}{\tau_p} = \left[1 - \frac{M}{M_P}\right]^{\frac{1}{4}} = [1 - 0.67]^{1/4} = 0.76$ for the $\sigma_{max} = F_y$ case

For Drucker: $\frac{\tau}{\tau_p} = [1 - 0.33]^{\frac{1}{4}} = 0.90$ for the $\sigma_{max} = F_y/2$ case

These values are close. However, Drucker cannot handle stress distributions where both moment and normal forces are acting. For the $\sigma_{max} = F_y$ case, a normal stress equal to $0.61\sigma_{max}$ used in combination with the von Mises formula gives the correct shear strength. For the $\sigma_{max} = F_y/2$ case, a normal stress equal to $0.57\sigma_{max}$ provides the right shear strength. Therefore, using von Mises with a normal stress of $0.6\sigma_{max}$ to calculate the average shear strength along the entire surface would be a reasonable approach for situations with stress reversal and $\sigma_{max} \leq F_y$.

Case 2: Net normal force, no Stress Reversal, $\sigma_{MAX} \leq F_y, \sigma_{MIN} = 0$



Ω_{vm} (σ_{Max}= F_y) =	1.00	0.99	0.97	0.94	0.89	0.84	0.76	0.66	0.53	0.31
Ω_{vm} (σ_{Max}= 0.50F_y) =	1.00	1.00	0.99	0.98	0.97	0.96	0.95	0.93	0.91	0.88

Figure A3. Case 2 von Mises shear strengths for each segment

Table A2. Net τ/τ_P

Case	von Mises
σ_{Max}= Fy	0.79
σ_{Max}= 0.5Fy	0.96

The reductions are the same for these cases when σ_{MIN} = -σ_{MAX} and there are no axial forces.

In each of the previous cases, using the von Mises equation with a normal stress of 0.6 σ_{MAX} gives good results.

For case where $\sigma_{MAX} = F_y$; 0.6 $\sigma_{MAX} = 0.6\ F_y$

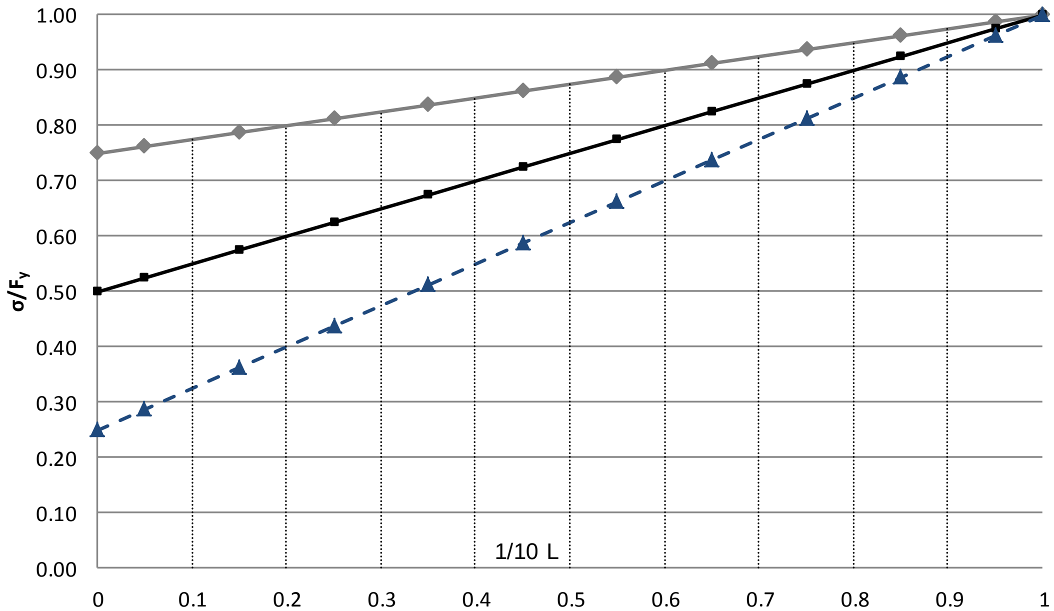
$$\frac{\tau_N}{\tau_p} = \left[1 - \left(\frac{0.6F_y}{F_y}\right)^2\right]^{\frac{1}{2}} = 0.80 \cong 0.79 \text{ from strip model}$$

For case where $\sigma_{MAX} = \frac{F_y}{2}$; 0.6 $\sigma_{MAX} = 0.3\ F_y$

$$\frac{\tau_N}{\tau_p} = \left[1 - \left(\frac{0.3F_y}{F_y}\right)^2\right]^{\frac{1}{2}} = 0.95 \cong 0.96 \text{ from strip model}$$

Case 2 is essentially one extreme of Case 1. Therefore, it is appropriate for the same approximate approach to provide reasonable values.

Case 3: Net normal force, no Stress Reversal, $\sigma_{MAX} \le F_y, \sigma_{MIN} > 0$



Ω_{vm} (σ_{Max}=0.75F_y) =	0.96	0.93	0.90	0.86	0.81	0.75	0.67	0.59	0.46	0.28
Ω_{vm} (σ_{Max}= 0.50F_y) =	0.85	0.81	0.78	0.73	0.68	0.63	0.56	0.47	0.37	0.20
Ω_{vm} (σ_{Max}= 0.25F_y) =	0.65	0.61	0.59	0.54	0.51	0.46	0.41	0.34	0.28	0.14

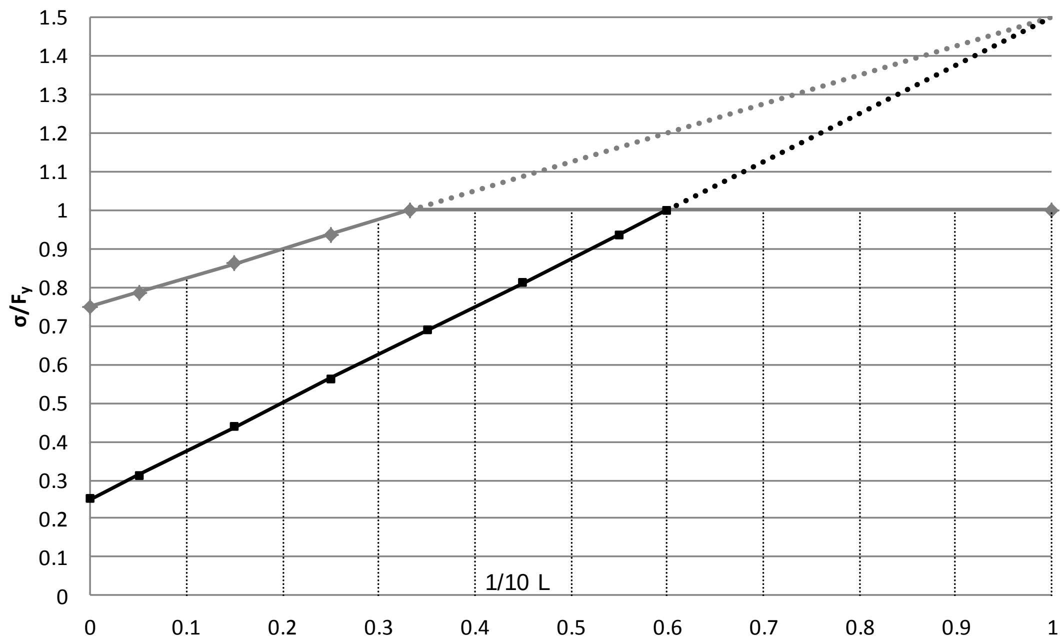
Figure A3. Case 3 von Mises shear strengths for each segment

For each case, the location along the surface at which the normal stress in combination with von Mises gives the identical net τ/τ_P value is as follows:

- For σ_{min} = 0.25F_Y: the normal stress at 0.59L from σ_{min} gives the correct net τ/τ_P value
- For σ_{min} = 0.50F_Y: the normal stress at 0.58L from σ_{min} gives the correct net τ/τ_P value
- For σ_{min} = 0.75F_Y: the normal stress at 0.56L from σ_{min} gives the correct net τ/τ_P value

Using von Mises with the normal stress at 0.6L from σ_{min} gives a reasonable value for τ/τ_P. This is consistent with the approach used when there is a stress reversal. The only difference is that, in this case, we use σ_{min} + 0.6 (stress difference) rather than 0.6σ_{max}.

Case 4: Net normal force, no Stress Reversal, $\boldsymbol{\sigma_{MAX} \geq F_y, \sigma_{MIN} > 0}$



Ω_{vm} (σ_{Max}= 0.75F_y) =	0.95	0.90	0.83	0.72	0.59	0.34	0.00	0.00	0.00	0.00
Ω_{vm} (σ_{Max}=0.25F_y) =	0.61	0.51	0.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure A4. Case 4 von Mises shear strengths for each segment

For each case, identify the length of segment, L_{seg}, of the surface where $\sigma \leq F_y$. Use the von Mises relationship evaluated at 0.6*L_{seg} to determine the available shear stress. Multiply result by the proportional length of segment ($\frac{L_{seg}}{L}$).

For the case where $\sigma_{min} = 0.75\, F_y$

$$L_{seg} = L\left[1 - \frac{1.5F_y - F_y}{1.5F_y - (0.75F_y)}\right] = 0.33\text{ L}$$

Determine normal stress at 0.6*L_{seg}

$$0.6L_{seg} = 0.6 * 0.33L = 0.20L$$

$$\sigma_{0.6} = 0.20L * \frac{(\sigma_{MAX} - \sigma_{MIN})}{L} + \sigma_{MIN} = 0.9F_y$$

Evaluating the von Mises relationship at 0.6*L_{seg}:

$$\frac{\tau_N}{\tau_p} = \left[1 - \left(\frac{0.9F_y}{F_y}\right)^2\right]^{\frac{1}{2}} * \frac{L_{Seg}}{L} = 0.14$$

For the case where $\sigma_{min} = 0.25\ F_y$

$$\mathrm{L}_{seg} = \mathrm{L}\left[1 - \frac{1.5F_y - F_y}{1.5F_y - (0.25F_y)}\right] = 0.60\ \mathrm{L}$$

Determine normal stress at 0.6*L_{seg}

$$0.6\mathrm{L}_{seg} = 0.6 * 0.60\mathrm{L} = 0.36\mathrm{L}$$

$$\sigma_{0.6} = 0.36L * \frac{(\sigma_{MAX} - \sigma_{MIN})}{L} + \sigma_{MIN} = 0.7F_y$$

Evaluating the von Mises relationship at 0.6*L_{seg}:

$$\frac{\tau_N}{\tau_p} = \left[1 - \left(\frac{0.7F_y}{F_y}\right)^2\right]^{\frac{1}{2}} * \frac{L_{Seg}}{L} = 0.43$$

Case 4 illustrates that two steps are required to determine the available shear stress

1. $\sigma_{0.6} = 0.6\left[1 - \frac{\sigma_{MAX} - F_y}{\sigma_{MAX} - \sigma_{MIN}}\right](\sigma_{MAX} - \sigma_{MIN}) + \sigma_{MIN}$
2. $\frac{\tau_N}{\tau_p} = \left[1 - \left(\frac{\sigma_{0.6}}{F_y}\right)^2\right]^{\frac{1}{2}} \left[1 - \frac{\sigma_{MAX} - F_y}{\sigma_{MAX} - \sigma_{MIN}}\right]$

A similar strip model evaluation was used to determine how to handle situations involving stress reversal and $\sigma_{max} > F_Y$. In this case:

$\frac{\tau_N}{\tau_p} = \left[1 - \frac{\sigma_{MAX} - F_y}{\sigma_{MAX} - \sigma_{MIN}}\right] 0.76$; as long as $|\sigma_{MIN}| < F_y$; if this is not true, the plate is overstressed

Summary

	Stress Reversal	No Stress Reversal
$\sigma_{MAX} \leq F_y$	(1) Use von Mises based on $\sigma = 0.6\ \sigma_{MAX}$	(2) Use von Mises based on σ @ 0.6 L from σ_{MIN} end, i.e., use $\sigma = \sigma_{MIN} + 0.6\ (\sigma_{MAX} - \sigma_{MIN})$
$\sigma_{MAX} > F_y$	(3) Use an Ω equal: $\left[1 - \frac{\sigma_{MAX} - F_y}{\sigma_{MAX} - \sigma_{MIN}}\right] 0.76$ where $\|\sigma_{MIN}\| < F_y$	(4) Use von Mises based on $\sigma = 0.6\left[1 - \frac{\sigma_{MAX} - F_y}{\sigma_{MAX} - \sigma_{MIN}}\right](\sigma_{MAX} - \sigma_{MIN}) + \sigma_{MIN}$; and then multiply the result by $\left[1 - \frac{\sigma_{MAX} - F_y}{\sigma_{MAX} - \sigma_{MIN}}\right]$

EXAMPLES

Example 1

30-in. by 1/2-in. plate; P = 150 kip; M = 2400 k-in.; F_y = 50 ksi

$$M_p = Z\,F_y = \frac{(1/2\text{in.})\,(30in.)^2}{4} \times 50\ ksi = 5625\ k\text{–}in.$$

$$f_b = \frac{M}{S} = \frac{2400\ k\text{–}in,}{\left[\frac{(30in.)^2\,(1/2\text{in.})}{6}\right]} = 32\ ksi$$

$$f_a = \frac{P}{A} = \frac{150\ kip}{30in.(1/2\text{in.})} = 10\ ksi$$

$$\sigma_{MAX} = 32\ ksi + 10\ ksi = 42\ ksi < F_y$$

$f_a < f_b$ (i.e. stress reversal)

$$\frac{\tau_N}{\tau_p} = \left[1 - \left(\frac{0.6\ (42\ ksi)}{50\ ksi}\right)^2\right]^{\frac{1}{2}} = 0.86$$

Example 2

Same plate as (1); P = 300 kip, M = 1000 k-in.

$$f_b = \frac{M}{S} = \frac{1000\ k\text{–}in.}{\frac{(30in.)^2\,(1/2\text{in.})}{6}} = 13\ ksi$$

$$f_a = \frac{P}{A} = \frac{300\ kip}{30in.(1/2\text{in.})} = 20\ ksi$$

$$\sigma_{MAX} = 13\ ksi + 20\ ksi = 33\ ksi < F_y$$

$f_a > f_b \Rightarrow$ no stress reversal

$$\sigma_{MIN} = 20\ ksi - 13\ ksi = 7\ ksi$$

$$\sigma_{0.6\ \mathrm{L}} = 7\ ksi + 0.6\ (33\ ksi - 7\ ksi) = 23\ ksi$$

$$\frac{\tau_N}{\tau_p} = \left[1 - \left(\frac{23\ ksi}{50\ ksi}\right)^2\right]^{\frac{1}{2}} = 0.89$$

Example 3

Same plate as (1); P = 300 kip, M = 3000 k-in.

$$f_b = \frac{M}{S} = \frac{3000\ k-in.}{\frac{(30in.)^2\ (1/_2\text{in.})}{6}} = 40\ ksi$$

$$f_a = \frac{P}{A} = \frac{300\ kip}{30in.(1/_2\text{in.})} = 20\ ksi$$

$$\sigma_{MAX} = 60\ ksi > F_y$$

$\sigma_{MIN} = -20\ ksi$ (reversal)

$$\frac{\tau_N}{\tau_p} = \left[1 - \frac{60\ ksi - 50\ ksi}{60\ ksi - (-20\ ksi)}\right] 0.76 = 0.67$$

Example 4

Same plate as (1); P = 500 kip, M = 2000 k-in.

$$f_b = \frac{M}{S} = \frac{2000\ k-in.}{\frac{(30in.)^2\ (1/_2\text{in.})}{6}} = 27\ ksi$$

$$f_a = \frac{P}{A} = \frac{500\ kip}{30in.(1/_2\text{in.})} = 33\ ksi$$

$$\sigma_{MAX} = 33\ ksi + 27\ ksi = 60\ ksi > F_y$$

$\sigma_{MIN} = 33\ ksi - 27\ ksi = 6\ ksi$ no reversal

Step 1: $\sigma_{V-M} = 0.6\left[1 - \frac{60\ ksi - 50\ ksi}{60\ ksi - 6\ ksi}\right](60\ ksi - 6\ ksi) + 6\ ksi = 32\ ksi$

Step 2: $\frac{\tau_N}{\tau_p} = \left[1 - \left(\frac{32\ ksi}{50\ ksi}\right)^2\right]^{\frac{1}{2}} \left[1 - \frac{60\ ksi - 50\ ksi}{60\ ksi - 6\ ksi}\right] = 0.63$

APPENDIX B

Buckling Considerations

Dimensions needed to determine the buckling capacities of corner “spans” are shown in the figures below. Dimensions “b” for both the vertical surface and the horizontal surface are the lengths of these surfaces as defined by the Basic Corner Check. Effective span lengths (a_h and a_v) extend orthogonally from the respective surface to the nearest fastener of the adjacent truss member. The span with the shorter buckling length is controlled by sidesway buckling and requires dimensions L_{s1} and L_{s2}. Both dimensions measure the distance between fasteners and member edges in a direction that is parallel to the corresponding “a” dimensions. L_{s1} is measured from the intersection of the centerline of the diagonal member and the row of fasteners nearest the work point, to the nearest edge of the adjacent member. L_{s2} is the distance between the intersection of the centerline of diagonal member and the end of member, to the nearest fastener of the adjacent member.

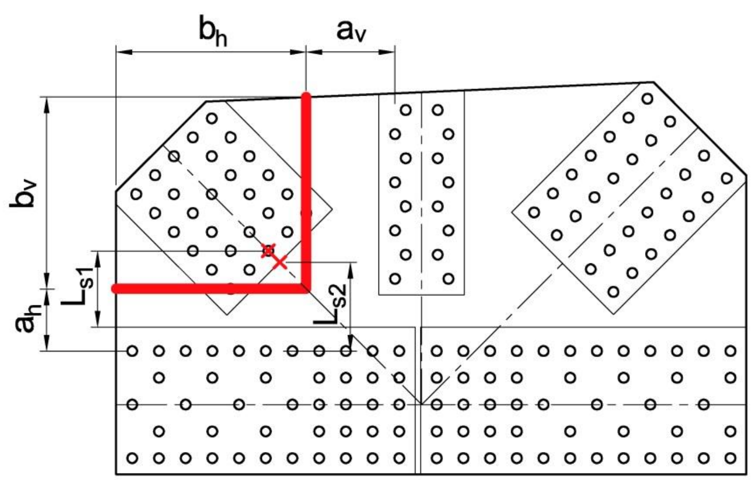


Figure 4. Example buckling length determination (a_h < a_v).

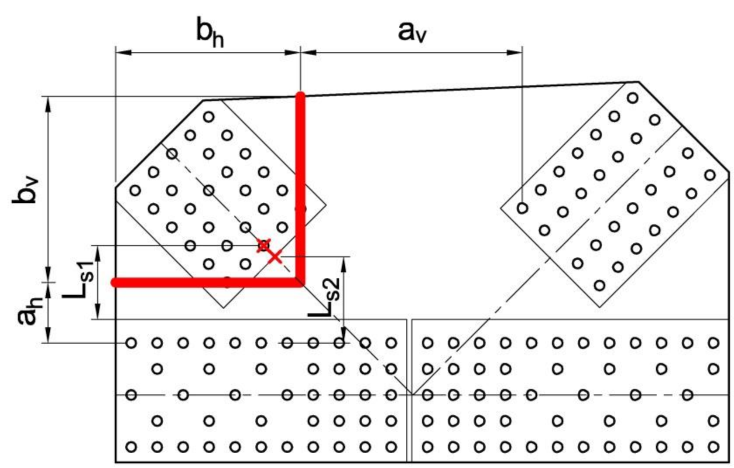


Figure 5. Example buckling length determination (a_h < a_v).

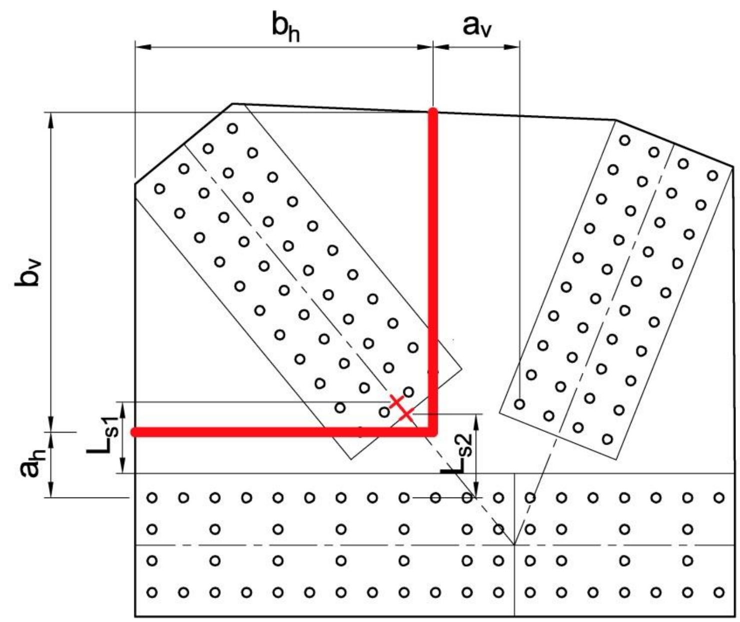


Figure 6. Example buckling length determination ($a_h < a_v$).

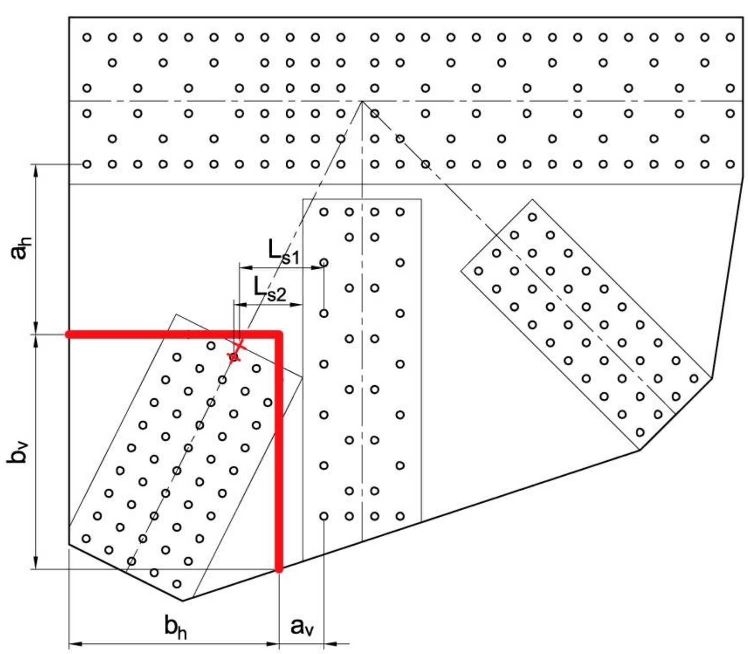


Figure 7. Example buckling length determination ($a_v < a_h$).

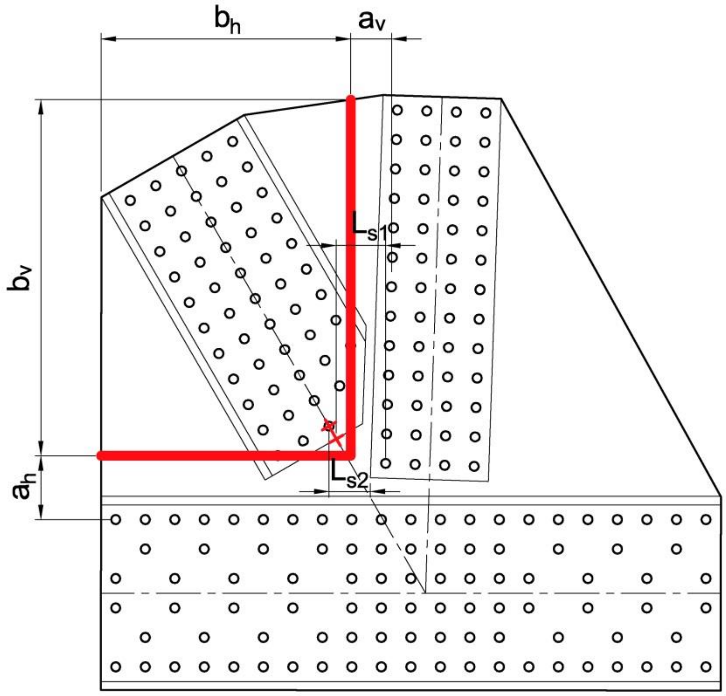


Figure 8. Example buckling length determination (a_v < a_h).

APPENDIX C

Relative Reliability Considerations

Background

The following equation forms the basis of the relative reliability approach that was used to develop the current AISC steel design specifications:

$$\beta = \frac{\ln(R_m/Q_m)}{[V_R^2 + V_Q^2]^{0.5}} \qquad \text{(Eq. 1)}$$

Where:

- β is the Relative Reliability Index
- R_m is the average strength of the elements being designed or evaluated
- Q_m is the average peak demand (i.e., load effect) for the return period in question
- V_R is the coefficient of variation (COV) of the distribution of actual strengths
- V_Q is the COV of the distribution of peak demands (i.e., load effects) for the selected return period

When the capacity and load distributions are ln-normal, the distributions of their natural logarithms are normal; and, the COV for each population represents the standard deviation of the population of the natural logarithms. In this case, the variable "ln(R/Q)" is also normally distributed, with a mean value approximately equal to the numerator in Eq. 1, and a standard deviation approximately equal to the denominator in Eq. 1. With this in mind, the β term is simply the number of standard deviations between the average ln(R/Q) value and zero. A value of zero for ln(R/Q) corresponds to a situation where R = Q. Consequently, it represents a condition of imminent failure (i.e. a situation where demand equals capacity), and β is a measure of the fraction of structural elements in a ln-normal population defined by R_m and V_R that would be expected to fail in a ln-normal demand environment defined by Q_m and V_Q.

While Eq. 1 is relatively simple, its application to structural issues is rather complicated. One primary reason for this is the fact that there are many factors that contribute to the variability in both capacity and demand, and quantifying these factors and their relationships to one another, is extremely difficult. In order to make application of Eq. 1 feasible without also making it meaningless, those responsible for developing the current AISC LRFD Steel Design Specifications (AISC Specs) made several assumptions. Among them was the assumption that all of the various sources of capacity and demand variability are statistically independent of one another. This assumption has the following two main consequences:

1. The COV for R (or Q) can be calculated as the square-root-sum-of-the-squares (SRSS) of the COVs for each variable factor that determines R (or Q). For example, if there are 3 properties that determine the capacity of a structural member - Property 1, Property 2 and Property 3 - then $V_R = [V_{P1}^2+V_{P2}^2+V_{P3}^2]^{0.5}$.
2. The resulting standard deviation of ln(R/Q) is an upper bound value, likely overestimating the actual standard deviation by a considerable degree. When factors that are the source of variability in R or Q are not really statistically independent, then combining their COVs via the SRSS method often overestimates V_R or V_Q.

Due to the second of these two consequences, resulting ln(R/Q) distributions should not be used to calculate actual probabilities of failure in populations of structural elements. In other words, β should not be used in conjunction with the normal probability distribution function to calculate a probability of failure (i.e., area under the curve to the left of zero). Instead, β should be considered a measure of relative reliability among populations with similar characteristics that are subjected to similar demands.

The development of the AISC Specs included substantial efforts to compile data and related statistics so that strength and demand distributions could be quantified as needed for application of Eq. 1. One offshoot of this work was the calculation of β values that were achieved by applying past and contemporary Allowable Stress (ASD) methods. These "historical" β values were used to develop benchmarks to be achieved by the AISC Load and Resistance Factor Design (LRFD) methods being developed. In general, current AISC LRFD design specifications have been developed so as to achieve β values in the range of 2.5 to 3.5 for structural members, and β values of at least 4 for connectors, when subjected to typical dead and live load combinations.

The current AASHTO MBE provisions are based on achieving/maintaining β values of about 3.5. This benchmark does not appear to have been based on a comprehensive study of the relative reliabilities provided by historical methods of gusset plate design. It is worth noting that the AASHTO method for calculating β is different than the method used by AISC. The AASHTO method is based on the assumption that R and Q are normally distributed (rather than ln-normally distributed). This means that the combined variable is "R-Q" rather than ln(R/Q). For the ranges of R and Q COV values used by AISC, the two approaches will yield similar results. When the COV for R is significantly higher than the COV for Q, the assumption that resistance follows a normal distribution can provide significantly lower β values.

In a conventional LRFD approach, R_m is related to Q_m as determined via the following:

$$R_m = R_N \times M \times F \times PF \qquad \text{(Eq. 2)}$$

Where:

- R_N is the nominal strength (strength when actual properties match nominal values)
- M is a material property bias factor = actual value/nominal value
- F is a fabrication-related bias factor representing the ratio of as fabricated properties to specified (nominal) properties
- PF is the "professional factor," which is the ratio of actual strength to calculated strength

and,

$$\phi \times R_N = LF \times Q_{des} \qquad \text{(Eq. 3)}$$

Where:

- ϕ is the LRFD strength reduction factor
- LF is the LRFD load factor
- Q_{des} is the total design service demand (i.e. the demand caused by application of D and L)

and,

$$Q_m = Q_{des} \times BIAS_Q \qquad \text{(Eq. 4)}$$

Where:

- $BIAS_Q$ is the ratio of Q_m/Q_{des} ($BIAS_Q$ is discussed below) so,

$$(\phi \text{ x } R_m)/(M \text{ x } F \text{ x } PF) = LF \text{ x } Q_m/BIAS_Q \quad \text{(Eq. 5)}$$

which means,

$$R_m/Q_m = (LF \text{ x } M \text{ x } F \text{ X } PF)/(\phi \text{ x } BIAS_Q) \quad \text{(Eq. 6)}$$

The FHWA Study used a value of 1.1 for M, 1.0 for F, and values of PF representing the ratios of strengths based on FE analyses divided by strengths calculated using simplified methods. This treatment of PF assumes the FE-based capacities are the same as actual capacities.

The FHWA study assumed variations in R were due to three factors; variability introduced via simplified calculations (V_{PF}), variation inherent in material properties (V_M), and variation introduced via fabrication (V_F). Constant values of 0.11 and 0.05 were used for V_M and V_F, respectively. V_{PF} values were determined from the distribution of PF values.

For Q, the FHWA study simply used dead load (D) and live load plus impact (L) bias and COV values taken from another report. The selected values were:

- D: bias = 1.05; COV = 0.10
- L: bias = 1.15; COV = 0.12

A particular case from the FHWA study will be used as an example of how these parameters can be used to quantify relative reliability. The D/L ratio will be 3.0 (i.e. D represents 75% of the service load, L represents 25%). In this case, Q_m is related to the total design demand (Q_{des}) as follows:

$$Q_m = [0.75(1.05) + 0.25(1.15)] \text{ x } Q_{tot} = 1.075\ Q_{des} \quad \text{(Eq. 7)}$$

So; $BIAS_Q$ = 1.075 in this case

Factored demand (Q_F) is related to Q_{des} as follows:

$$Q_F = [0.75(1.25) + 0.25(1.75 \text{ x } 1.33)] \text{ x } Q_{des} = 1.52\ Q_{des} \quad \text{(Eq. 8)}$$

So; the net LF is 1.52 in this case

Eliminating Q_{des} we get:

$$Q_F = 1.41\ Q_m \quad \text{(Eq. 9)}$$

Regarding the variability of Q; V_Q is a function of the variability of both D (V_D = 0.10) and L (V_L = 0.12), which can be estimated as follows[1]:

$$V_Q = [0.75^2(0.10)^2 + 0.25^2(0.12)^2]^{0.5} = 0.081 \quad \text{(Eq. 10)}$$

On the capacity side of this example, we will be checking a plate for Whitmore buckling. According to the FHWA Study, the bias (PF) and bias-related COV (V_{PF}) inherent in the Whitmore capacity check are

[1] The variability associated with the demand at a particular point in a structure (e.g., Whitmore compression at the end of a

1.24 and 0.127, respectively, and ϕ for Whitmore buckling at a D/L of 3.0 is about 0.93 (FHWA Report Figure 74). Putting these and previously given values into Eq. 6 gives:

$$R_m/Q_m = (LFxMxFxPF)/(\phi \text{ x } BIAS_Q) = (1.52x1.1x1x1.24)/(0.93x1.075) = 2.07 \quad \text{(Eq. 11)}$$

Regarding the variability of R; the FHWA Study assumed V_R is a function of the variability of M, F and PF, which, as noted previously, have values of $V_M = 0.11$, $V_F = 0.05$ and $V_{PF} = 0.127$, respectively. The corresponding V_R is calculated as follows:

$$V_R = [0.11^2 + 0.05^2 + 0.127^2]^{0.5} = 0.175 \quad \text{(Eq. 12)}$$

We now have everything we need to calculate β for this example using Eq. 1:

$$\beta = \frac{\ln(R_m/Q_m)}{[V_R^2 + V_Q^2]^{0.5}} = \ln(2.07)/(0.081^2 + 0.175^2)^{0.5} = 3.77 \quad \text{(Eq. 13)}$$

Since the ϕ values used in this example were developed in order to achieve a β value of 3.5, a result close to this value was expected.

As previously noted, the relative reliability method used by the FHWA Study to establish ϕ factors was based on the assumption that load and resistance variables are normally distributed rather than ln-normally distributed. Since the variability in β is dominated by V_R in this example, we would expect the β value based on normal distributions would be considerably lower than 3.77. This is at odds with the fact that the FHWA Study reportedly assumed normal distributions, and established ϕ factors to provide β values of 3.5. Using the parameters previously defined, we can determine the values needed to calculate β assuming normally distributed load and resistance variables.

- $R_m - Q_m = 1.07Q_m$
- σ_R = standard deviation of resistance = 0.175 x $2.07Q_m = 0.362Q_m$
- σ_Q = standard deviation of demand = 0.081 x $Q_m = 0.081Q_m$

$$\beta = 1.07Q_m /[(0.081Q_m)^2 + (0.362Q_m)^2]^{0.5} = 2.88$$

As expected, this value is considerably lower than the value based on ln-normal distributions of R and Q. Its large deviation from the FHWA Study target value of 3.5 is not expected. This suggests that the Monte Carlo evaluation used as part of the FHWA Study did not randomly select load and Whitmore Buckling resistance values from normal distributions defined by the parameters used in this example. However, the distribution parameters used in this example were taken from the FHWA Study discussion related to loads and Whitmore Buckling.

Variation in β Inherent in the 2014 AASHTO Standards

Given the BACKGROUND discussion, it should be clear that the MBE ϕ values are based on specific values of PF that, in many cases, are significantly greater than 1.0, and values of V_{PF} that are significantly greater than zero. This means, each MBE ϕ value was developed to be used with a capacity calculation method that has a particular bias and degrees of variability. Put another way, the manner in which the MBE ϕ values were developed ostensibly requires users of other capacity calculation methods to use different ϕ factors.

Since there are usually many valid methods for calculating the capacities of structural elements, making ϕ factors method-dependent can create problems. The MBE provisions themselves can be used to demonstrate this fact.

According to the MBE, the engineer has the option of using a finite element (FE) model to determine gusset plate capacities. If we assume that the engineer creates a perfect FE model for every case (the best possible application of the alternative approach), the associated PF (bias) will be 1.0, and V_{PF} will be zero. Using the previous gusset plate buckling example, and replacing the Whitmore bias and V with the ideal FE values of 1.0 and zero, we get the following:

$V_Q = [0.75^2(0.10)^2 + 0.25^2(0.12)^2]^{0.5} = 0.081$ (as before) (Eq. 14)

$V_R = [0.11^2 + 0.05^2 + 0.0^2]^{0.5} = 0.121$ (Eq. 15)

$R_m/Q_m = (LF \text{ x } M \text{ x } F \text{ x } PF)/(\phi \text{ x } BIAS_Q) = (1.52 \text{x} 1.1 \text{x} 1 \text{x} 1.0)/(0.93 \text{x} 1.075) = 1.67$ (Eq. 16)

$$\beta = \frac{\ln(R_m/Q_m)}{[V_R^2 + V_Q^2]^{0.5}} = \ln(1.67)/(0.081^2 + 0.121^2)^{0.5} = 3.52 \quad \text{(Eq. 17)}$$

The relative reliability decreases when a perfect capacity determination method is substituted for the standard approach. This is an interesting artifact of the manner in which the MBE provisions were developed. It runs counter to the notion that, if an improved capacity calculation method is used, uncertainty and variability in the resulting capacities will decrease, and higher values of ϕ should be used. At the very least, this example shows that strict adherence to a particular degree of relative reliability is unrealistic. If we carefully dissected the load side of the reliability “equation,” we would find even more examples of the variable nature of reliability inherent in the current standards. This is not due to any errors in their development. It is simply due to the inability of the selected methods to consistently account for the many sources of variability inherent in the construction industry. All current structural engineering standards provide varying degrees of reliability.

Let’s consider a more practical example of an alternate capacity calculation method in which the bias is 1.1, and the COV is 0.1. In this case, $R_m/Q_m = 1.84$, $V_R = 0.157$, and $\beta = 3.45$. This is within 2 percent of the β achieved using a truly perfect FE model, which means it is clearly a reasonable substitute for the approaches explicitly accepted by the MBE.

The dependence of ϕ on PF could be avoided by calculating ϕ using a PF of 1.0 and a V_{PF} of zero. If this were done, the resulting ϕ factors would be suitable for use with a perfect capacity determination method, and any imperfect method that is known to err on the conservative side. This is consistent with the general engineering approach of applying conservatism in proportion to the degree of uncertainty or ignorance that exists.

Relationship Between ϕ and D/L Ratio

The current AASHTO Strength I load factor for dead load (D) is 1.25, and the associated live load (L) load factor is between 1.75 and 2.33. According to a reference included in the FHWA Study, the COV for D (V_D) is 0.10, and the COV for L (V_L) is 0.12. Given these assumptions, it is easy to see why ϕ factors must decrease as the D/L ratio increases in order to maintain a constant β. In a general sense, the load factor (LF) should be proportional to V. A load effect with low variability should have a low LF (near 1.0), while a load effect with higher variability, should have a higher LF. Load effects with similar variability should have similar LFs. As noted above, D and L are assumed to exhibit similar degrees of variability, while the respective load factors are substantially different. This is the source of the problem.

As D/L increases, the net LF decreases because LF_D is much smaller than LF_L. While the net variability in the overall load also decreases (V_D is smaller than V_L), it does so at a much slower rate. As a result, β, which is a measure of the ratio of net LF/net V, also decreases. To keep β constant, the FHWA study noted that ϕ must vary as D/L varies. However, since ϕ is supposed to be proportional to variability on the strength side, it really should not be affected by load-related issues. Assuming the COV values used by the FHWA Study accurately represent variability in load effects (a questionable assumption as noted in Footnote 1), the appropriate solution to this issue would be to modify the D and L load factors to be consistent with their respective degrees of variability,.

To put this into perspective, re-arrange Eq. 1 to get R_m/Q_m isolated as shown below:

$$R_m/Q_m = \exp[\beta(V_R^2+V_Q^2)^{0.5}] \qquad \text{(Eq. 18)}$$

In this case, the term on the right is essentially the "safety factor" that is needed in order to provide a relative reliability equal to β. This "safety factor" is comparable to an LRFD LF divided by ϕ. To make this simple, consider a situation in which the strength side of the equation is perfectly defined (i.e., there is no bias and V_R is zero). In this case, Eq. A1 becomes:

$$R_m/Q_m = \exp[\beta \text{ x } V_Q] \qquad \text{(Eq. 19)}$$

For the values used in the FWHA study (i.e., $\beta = 3.5$, $V_D = 0.10$, $V_L = 0.12$), the dead-load-only safety factor (i.e., the R_m/Q_m value when $\beta = 3.5$ and $V_Q = 0.10$) would be 1.42, and the live-load-only safety factor (i.e., the R_m/Q_m value when $\beta = 3.5$ and $V_Q = 0.12$) would be 1.52. In this case, for any given ϕ, LF_L is only 7% bigger than LF_D. Yet, according to AASHTO, LF_L is 40 to 86 percent greater than LF_D. Clearly, if we relate the load factors to the degree of variation exhibited by the loads (which is consistent with the purpose of LFs), LF_D and LF_L should be much closer (assuming of course the assumptions regarding V_D and V_L are correct). If this were done, there would be no need to vary ϕ as D/L varies.

As noted in a footnote related to the variability of load effects, the development of the current MBE β and ϕ values did not consider variability introduced by the process of transferring loads into load effects (V_T). Using the V_T value for L that was used by AISC (0.20), V_Q becomes 0.23. In this case, the live-load-only LF for a β of 3.5 would be 2.24. This is much more consistent with the current AASHTO Strength I load factor for L. Use of this higher V_Q in the FHWA study would have made ϕ much less sensitive to D/L. It also would have resulted in lower, and likely more accurate, β values.